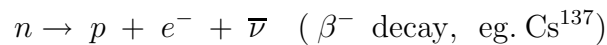## BETA DECAY

Introduction

   Beta particles are positrons and electrons released in a weak nuclear decays. The study of beta-decay spectra led to discovery of the neutrino. This experiment is concerned with the beta decays of Cs^{137} and Na^{22}. In this experiment, you will use the Cs^{137} decay to calibrate the instrument and determine some of its characteristics and you will make a Fermi-Kurie plot of the Na^{22} beta spectrum to demonstrate Fermi's theory of beta decay and confirm the small mass of the neutrino. Parity and charge-symmetry violation are also studied using beta decay, but not in this lab.

Theory

   A beta decay can involve either of the following two reactions:

$$n \rightarrow p + e^- + \overline{\nu} \quad (\ \beta^- \ \text{decay}, \ \text{eg.} \ Cs^{137})$$

$$p \rightarrow n + e^+ + \nu \quad (\ \beta^+ \ \text{decay}, \ \text{eg.} \ Na^{22})$$

   The total kinetic energy released in a beta decay, E_0, is the difference in rest energy between the initial nucleus and the end-products; it is typically on the order of 1 MeV. When performing calculations of the beta decay spectrum, you should assume that the recoil kinetic energy of the nucleus is negligible. Furthermore, your data should justify the conclusion that the rest mass of neutrino is negligible.

   To calculate the number of beta decays resulting in electrons with kinetic energy E_e and neutrinos with kinetic energy $E_o - E_e$, we use Fermi's Golden Rule for the transition rate between a bound state and a continuum of states:

$$w = \frac{2\pi}{\hbar} |M_{ij}|^2 \rho_f \quad \text{transition probability/time}$$

where $|M_{if}|$ is the matrix element describing the decay and ρ_f is the density of states near the final state. For a given decay, $|M_{if}|$ is constant (see Eisberg, Jelley, or Wong), so that the observed spectrum is proportional to the density of final states. When computing the density of final states, it is necessary to consider the electrical interaction between the beta particle and the electric field of the nucleus. The Fermi factor, $F(Z, p)$, is a function of the electric charge of the nucleus, Z, and the momentum of the emitted electron, p. It is computed in the program BETA.C that accompanies this lab.[1]In the old days, $F(Z, p)$ was found

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[1]†

with the *Tables for the Analysis of Beta Spectra* produced by the National Bureau of Standards. The notes on beta spectra that accompany the table are excellent. If $N(p)$ is the probability that an electron is emitted with momentum between p and $p + dp$, then, if the neutrino is massless we have:

$$N(p) = C\, F(Z, p)\, p^2 (E_0 - E_e)^2$$

where C is a collection of constants. The derivation is described in detail in Jelley and in Wong.

If $[N(p)/p^2 F(Z, p)]^{1/2}$ is plotted against E_e the result should be a straight line with x–intercept E_0. This is known as a Fermi-Kurie plot. A linear Fermi-Kurie plot indicates that Fermi's theory of beta decay is correct.

In Fermi's theory it is assumed that the emitted electron or positron has no orbital angular momentum. This is not always true (see Wong, pp. 212-213 or Jelley, pp. 93-99). Decays not predicted by Fermi's theorem are called unallowed or forbidden decays. Fermi-Kurie plots of forbidden decays are not linear.

Experimental Setup

The Magnetic Lens Spectrometer

A schematic of the experimental is shown in Figure 1. The source is placed at one end of a long evacuated tube. A vacuum is maintained in the chamber so that electrons emitted in beta decay are less likely to scatter off of air molecules. Magnetic coils create a nearly uniform magnetic field inside the chamber that points along its axis. The effect of this field is to cause any particle whose initial trajectory was skewed from the axis of the spectrometer to follow approximately a helical path down the length of the tube. By using a series of baffles, one selects a group of particles within a small range of momenta by varying the magnetic field. The particles are focused on a detector at the far end of the tube. The entire system is called a magnetic lens (for a more detailed explanation, see Halliday §6.6 and Seward).

Calibration of the Spectrometer

As described above, the spectrometer selects electrons with momenta defined by the strength of the magnetic field. You can measure the current producing the magnetic field. On the wall next to the apparatus is a chart of magnetic field strength at 20 A at different locations inside the spectrometer. Note that the field is not constant in space. Since the relationship between I and B is linear, the relationship between I and p is linear, even if the magnetic field is not spatially uniform. The Cs^{137} spectrum has a reference peak that can be used to calibrate the spectrometer. The decay schemes for Na^{22} and Cs^{137} are as follows:
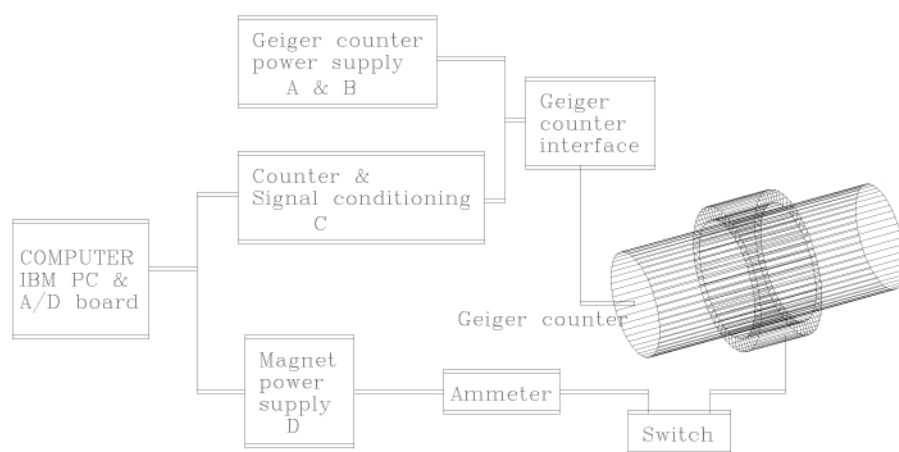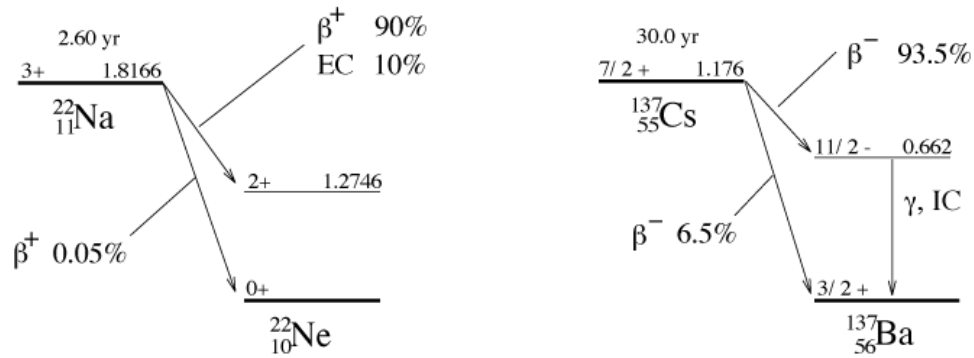
Figure 1: Schematic of the beta decay experiment.

Figure 2: For each nuclear state, reresented by a horizontal bar, the number on the left is the angular momentum, the sign is the parity of the nucleus, and the number on the right is the energy, in MeV, relative to the lowest energy in the scheme. EC is for electron capture. IC is for internal conversion. From Dan Walter '96

Note that 8% of the beta decays take the excited Cs^{137} nucleus to the ground state of Ba^{137}. This is a 1.18 MeV decay. (What does this tell you about the endpoint energy for this decay?) The other 92% of the beta decays take the nucleus to an excited state of Ba^{137}. The excited Ba^{137} nucleus decays to its ground state by emitting a gamma ray 90% of the time. The gamma rays are unaffected by the magnetic field and have to be blocked by lead bricks inside the spectrometer. The remaining 10% of the time, the nucleus decays (with the half life of about 2 minutes) by emitting an internal conversion electron. Thus, there will be a peak of known energy in the Cs beta decay spectrum. Its momentum can be determined by relativistic kinematics (see Halliday 5.13-5.14). Use this peak to calibrate the spectrometer. If the measured width is much larger than the natural line width, then the conversion peak can also be used to find the resolution of the spectrometer; it is the width at half maximum divided by the momentum at which the peak is centered. Keep in mind that because there are two beta decays and a conversion peak for Cs^{137}; the spectrum you see is superposition of all three.

Correction Fields

Because the magnetic lens depends upon the field inside the tube, stray magnetic fields might have an adverse effects on its characteristics. For this reason, the spectrometer is surrounded by a rectangular cage, the edges of which support magnetic "trim" coils. The purpose of these coils is to cancel magnetic fields from outside sources (most notably the Earth). To estimate the effects of the Earth's field, examine the Cs^{173} decay with the trim coils set to produce ≈ 1

Gauss magnetic field and compare to the results with the trim coils off. If the effect is substantial, find a suitable position for compasses so that they can point in the horizontal and vertical planes. Keep in mind that you should be as close to the axis of the cylinder as possible. Once you have positioned the compasses, adjust the current in the coils so that the field points along the axis of the spectrometer, and any components perpendicular to the axis of the cylinder are canceled. Then, quantify the effect of the coils.

Vacuum and windows

The cylinder of the spectrometer is evacuated by means of a mechanical pump in order to reduce the scattering of the beta rays. The vacuum is measured with a thermocouple gauge. See Melissinos pages 133-135 for an explanation. What is the mean free path between particles in the vacuum or the column depth along the tube in units of g/cm^2? How bad would the vacuum have to be to affect your answers? What would happen if the vacuum windows were thicker? How might you test this? They are made of 0.03 cm kapton with a density of about $1.4 \ g/cm^3$.

Particle Detector

The particles are detected by means of a thin ($1.4 \ mg/cm^2$) window Geiger Müller tube (Melissinos, pp 175-177, 183-188). The high voltage supply for the tube is on top of the electronics rack. It is set to 1200 Volts. The pulse out of the tube has an amplitude of -0.2 V and is about $2 \ \mu s$ wide, why? (look at it with a scope!). It is first amplified, then inverted, and finally the discriminator sets the voltage threshold for events in which we are interested in. Finally, the pulses are sent to a counter that is read by the computer. The knobs are preset for you.

Experimental Procedure

You will of course be using radioactive sources in this lab. The sources are built into their holders in such a way that their emission is strongly directional; avoid looking straight into them. Familiar yourself with the level of radioactivity. The Geiger counter reads $\approx 14,000$ counts/min when place 20 cm from the source. This corresponds to ≈ 3 mrem/hr for gamma radiation. One rem corresponds to ≈ 100 ergs/s of absorbed energy. The maximum safe operating levels in an unrestricted area, as set by the Atomic Energy Commission, are less than 2 mrem in one hour or 100 mrem in seven consecutive days or 500 mrem in one year. The average dose in the US in one year is 360 mrem; radon accounts for more than half of this.

When running the experiment, there are several electrical details to be aware of. First, when turning on the devices in Figure 1, throw the switches labeled A, B, C, D in the order A, C, D, B (A is a "standby power" switch). Secondly, there

is a switch on the magnet power supply labeled "int/ext". The switch should be left in the "int" position. This switch passes control of the current to the magnet to either an internal source (like you turning the knobs) or an external source (like the computer). While taking actual data the computer is doing most of the work, but you must tell it what to do. After turning the computer on, insert a $5\ 1/4$" Double Density (NOT High Density) floppy into the drive. Your data will be stored on it. Then,

1. Type BASICA at the prompt and then 'load' the program BETA2.

2. Type the run command and answer the queries. You want to tell the computer how much time to be spend counting at each setting of current to the magnet (this is divided into 5 second periods) and you will want to tell the minimum and maximum current values to send to the magnet.

3. After answering the queries, the computer will urge you to switch the power supply to 'ext' mode. Do it.

4. Next, turn the current and voltage limiting dials (labeled V and I on the power supply) up to nearly full scale. If these dials are set too low, the computer will try to send a current to the magnet that is above the limiting values you set. If this happens, you will not get any useful data.

5. Flip the large knife switch to the right so that current can be passed to the magnetic lens.

6. Press return and the computer will start taking data. The computer first sends a signal to the power supply of the magnetic lens telling it what the current should be. It then receives counts from the detector for the time period you have specified. At the end of the counting period, the computer increments the current by the amount that you have chosen and again begins counting. The program also plots the resulting counts versus current as the data taking progresses.

When the computer is finished, 1) Slowly turn the dials V and I back to zero; 2) Flip the large switch back to the center; 3) Flip the int/ext switch back to int.

7. When queried, give the name of the data file. It will be written to the diskett.

8. To leave BASICA, type 'SYSTEM.'

Because of the large currents and magnetic fields involved, there are a number of things that you must be careful about. You will want to make sure that you keep credit, bank and ID cards away from the magnet; they could potentially be erased. Also, **never** flip the large switch to the magnet during the date-taking or at any time that you see a current on the ammeter (or when V and I are not zero). Very large currents are passed to the magnetic lens and if the switch is thrown while 14 A are going through it, there will be a very large and dangerous spark. Also, **never** flip the int/ext switch back to int before turning the V and I knobs to zero.

Doing this would cause a huge and damaging spike of current to be passed to the spectrometer.

Data analysis

The data analysis for this lab is done with an eye toward confirming Fermi's theory of beta decay and establishing the small mass of the neutrino. Toward that end, your reports should include a counts vs. momentum and a counts vs. energy plot for both Cs^{137} and Na^{22} for the full energy range of the spectrometer. For Na^{22} you should show the Fermi-Kurie plot, from which the value of E_0 and an upper bound on the mass of the neutrino should be deduced. You should also interpret the major features of the plots.

There are a number of sources of systematic error in this experiment. Three of them are 1) scattering of beta particles by the residual air, 2) scattering by the vacuum windows and the face of the Geiger-Müller tube, and 3) the Earth's magnetic field. You should tackle one of these three and show quantitatively how it effect on the counts vs. energy plot for Cs (why do we choose Cs?).

To help with the analysis, Yang-Hui He ('96) has written a C program. The program may be obtained from L. Page. If you use the program, you are responsible for knowing how it works.

Questions to ponder

• Why is there a continuous distribution of beta particle energies? • If the neutrino had mass, what would the Fermi-Kurie plot look like? • What is a forbidden transition an what would it look like on a Fermi-Kurie plot? • Why is the K-conversion electron mono-energetic? • How does a Geiger-Müller tube work? • What are the trajectories of the beta particles? • What is the magnetic field inside the spectrometer when the current is near 10 amps? • Why shouldn't you open the knife switch when current is flowing? • Is the Fermi-Kurie plot useful for the Cs decay? • Why is the instrument response to the K-conversion electrons broad? What is that telling you about the instrument? What is the energy resolution of the instrument? • In the Cs decay, what does the ratio of the area under the K-conversion spike to the area under the "beta" peak tell you? • What is the absorption cross-section for beta particles vs energy when they traverse a piece of material? (Melissinos, pg 150 et seq.) • What is the error bar per point? • What is a statistically sound way to search for a neutrino mass with your data? • How susceptible is your result for the endpoint of the Na decay to background subtraction? • The sources have a strength on order $100 \ \mu C$; does this correspond to what you measured with the Geiger counter?

References

1.Seward, Princeton Senior Thesis (May 1953). A copy is in the Advanced Lab Library. Pages 9-10 and 21-29 are essential.

2.Halliday. Introductory Nuclear Physics. Call number SK8215.427.11. Sections 6.1 and 6.7-14 are on the general phenomena and analysis of the spectrum, though not very helpful. Section 6.6 is essential for understanding the spectrometer. Sections 5.13 and 5.14 are necessary for calibrating the spectrometer.

3.*Tables for the Analysis of Beta Spectra*, National Bureau of Standards. applied Mathematics Series 13. A copy is on the undergraduate Reserve Shelf. This gives an excellent presentation on how to make a Fermi-Kurie plot.

4.Wong. Introductory Nuclear Physics. Pages 207-233.

5.Jelley. Fundamentals of Nuclear Physics. Call number QC716.J45 1990. Chapter 3 (pages 77-111) is a complete description of Beta decay.

6. Eisberg. Fundamentals of Modern Physics. Call number 8220.322. Pages 620-640 contain a solid explanation of beta decay, especially Fermi's theory. Not as rigorous as Jelley or Wong (but shorter).

7. Mellissinos. Experiments in Modern Physics, Academic Press, Chapter 2, section 5.

Revised by L. Page, Sept. 1995