KNIGHTS AND SPIES: THE CHAIN BUILDING STRATEGY

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A chain of people, each of its members supporting the next person along, is a desirable configuration for a questioner to create. Any such chain consists of a number (possibly zero) of spies, followed by a number (again possibly zero) of knights. There are $m+1$ possible configurations for a chain of length m. Provided we have a knight to hand, its members can be identified using repeated bisection in a mere $\lfloor \log_2 m \rfloor + 1$ questions; this meets the theoretical minimum for binary questions. An example is shown in Figure 1 below.

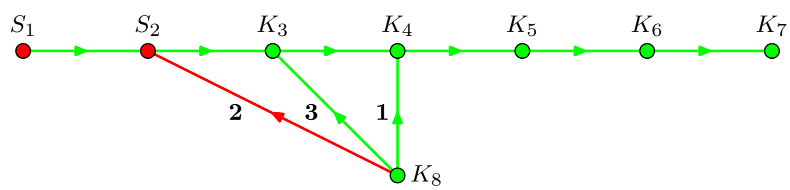

Figure 1. Person 8 is known to be a knight. Three questions to him suffice to find all identities in the chain formed by persons 1 to 7.

In this note we give a very rough outline of the *Chain Building Strategy*, in which these chains play a fundamental role. In the first step of the Chain Building Strategy we hunt for someone who we can guarantee is a knight by building chains, starting a new chain as soon as we meet an accusation. We then recursively link these chains by asking further questions, targeting people with the most persuasive support so far, and stop as soon as we reach someone who must be a knight. (The correct definition of 'most persuasive support' is slightly subtle and we shall not attempt to define it here.) In the second step we use this guaranteed knight to find everyone else's identity, saving questions by bisecting the existing chains as much as possible. An example of the critical first step is shown in Figure 2 overleaf.

Simulation—both by hand, and by computer—of the Chain Building Strategy strongly suggests that, provided the behaviour of the spies is constrained in some way, or randomised entirely, it never requires more than $n+\ell-1$ questions to find everyone's identity. Moreover, it appears to require on average about $4n/3$ questions to deal with a room in which knights are only just in the minority; this more than meets the requirements of Conjecture 5.1 in [1]. Sadly, it appears that

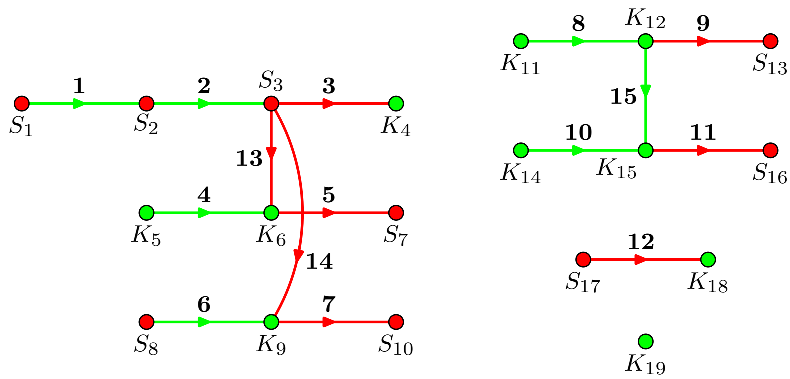

Figure 2. The question graph after the first step of the Chain Building Strategy in a room with 10 knights and 9 spies. Spies act knavishly, with the exception of S_8, who we suppose answers truthfully when asked about K_9. As in Step 1 of the Spider Interrogation Strategy, the components containing S_1 and S_{17} are disregarded once it becomes clear (after questions 14 and 12 respectively) that they contain at least as many spies as knights. The first step ends after question 15, after which we can be sure that person K_{15} is a knight. The first question in Step 2 will ask him for the identity of S_3, thereby bisecting the longest chain.

when ℓ is a smaller fraction of n, for example, $\ell = n/4$, the strategy is less effective. Figures 3 and 4 present some of the relevant data.

At the time of writing, these intermediate values for ℓ seem to present the largest obstacle in the path to a proof of Conjecture 5.1 in [1].

References

[1] M. Wildon, Knights, spies, games and ballot sequences. *To appear in Disc. Math.*

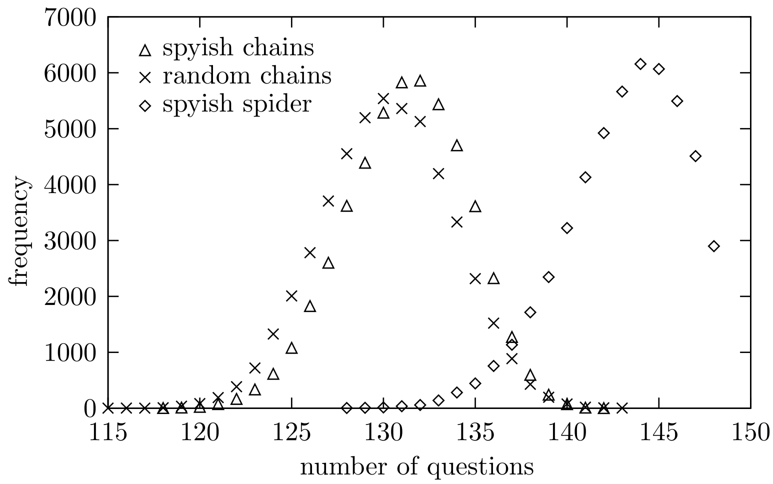

Figure 3. Numbers of questions asked in 25000 runs of the Chain Building Strategy in random generated rooms with 51 knights and 49 spies. Results for spyish spies, and spies which answer 'knight' or 'spy' at random with equal probabilities, are shown. For comparison, the corresponding results obtained from simulation of the Spider Interrogation Strategy with spyish spies are also shown.

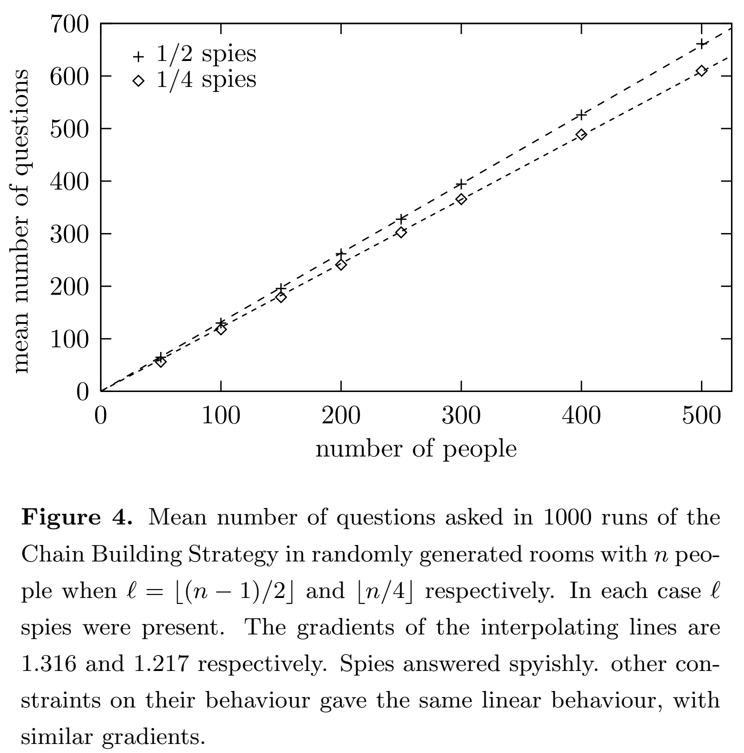

Figure 4. Mean number of questions asked in 1000 runs of the Chain Building Strategy in randomly generated rooms with n people when $\ell = \lfloor (n-1)/2 \rfloor$ and $\lfloor n/4 \rfloor$ respectively. In each case ℓ spies were present. The gradients of the interpolating lines are 1.316 and 1.217 respectively. Spies answered spyishly. other constraints on their behaviour gave the same linear behaviour, with similar gradients.