ADVANCED PROBLEMS AND SOLUTIONS

Edited by
Raymond E. Whitney

Please send all communications concerning ADVANCED PROBLEMS AND SOLUTIONS to RAYMOND E. WHITNEY, MATHEMATICS DEPARTMENT, LOCK HAVEN UNIVERSITY, LOCK HAVEN, PA 17745. This department especially welcomes problems believed to be new or extending old results. Proposers should submit solutions or other information that will assist the editor. To facilitate their consideration, all solutions should be submitted on separate signed sheets within two months after publication of the problems.

PROBLEMS PROPOSED IN THIS ISSUE

H-574 ***Proposed by José Luis Díaz-Barrero, University of Catalunya, Barcelona, Spain***

Let n be a positive integer greater than or equal to 2. Determine

$$\frac{F_n + L_nP_n}{(F_n - L_n)(F_n - P_n)} + \frac{L_n + F_nP_n}{(L_n - F_n)(L_n - P_n)} + \frac{P_n + F_nL_n}{(P_n - F_n)(P_n - L_n)},$$

where F_n, L_n, and P_n are, respectively, the n^{th} Fibonacci, Lucas, and Pell numbers.

H-575 ***Proposed by N. Gauthier, Department of Physics, Royal Military College of Canada***

Problem Statement: "Four Remarkable Identities for the Fibonacci-Lucas Polynomials"

For n a nonnegative integer, the following Fibonacci-Lucas identities are known to hold:

$$L_{2n+2} = 5F_{2n+1} - L_{2n};\;\; F_{2n+3} = L_{2n+2} - F_{2n+1}.$$

The corresponding identities for the Fibonacci $\{F_n(u)\}_{n=0}^{\infty}$ and the Lucas $\{L_n(u)\}_{n=0}^{\infty}$ polynomials, defined by

$$F_0(u) = 0,\; F_1(u) = 1,\; F_{n+2}(u) = uF_{n+1}(u) + F_n(u),$$
$$L_0(u) = 2,\; L_1(u) = u,\; L_{n+2}(u) = uL_{n+1}(u) + L_n(u),$$

respectively, are:

$$L_{2n+2}(u) = (u^2+4)F_{2n+1}(u) - L_{2n}(u);\;\; F_{2n+3}(u) = L_{2n+2}(u) - F_{2n+1}(u). \tag{1}$$

For m, n nonnegative integers, with the convention that a discrete sum with a negative upper limit is identically zero, prove the following generalizations of (1).

Case a: $$(2n+2)^{2m}L_{2n+2}(u) = (u^2+4)\left[\sum_{l=0}^{m}\binom{2m}{2l}(2n+1)^{2l}\right]F_{2n+1}(u) + u\left[\sum_{l=0}^{m-1}\binom{2m}{2l+1}(2n+1)^{2l+1}\right]L_{2n+1}(u) - [(2n)^{2m}]L_{2n}(u).$$

Case b: $$(2n+3)^{2m}F_{2n+3}(u) = \left[\sum_{l=0}^{m}\binom{2m}{2l}(2n+2)^{2l}\right]L_{2n+2}(u) + u\left[\sum_{l=0}^{m-1}\binom{2m}{2l+1}(2n+2)^{2l+1}\right]F_{2n+2}(u) - [(2n+1)^{2m}]F_{2n+1}(u).$$

Case c: $$(2n+2)^{2m+1}F_{2n+2}(u)=u\left[\sum_{l=0}^{m}\binom{2m+1}{2l}(2n+1)^{2l}\right]F_{2n+1}(u)$$
$$+\left[\sum_{l=0}^{m}\binom{2m+1}{2l+1}(2n+1)^{2l+1}\right]L_{2n+1}(u)-[(2n)^{2m+1}]F_{2n}(u).$$

Case d: $$(2n+3)^{2m+1}L_{2n+3}(u)=u\left[\sum_{l=0}^{m}\binom{2m+1}{2l}(2n+2)^{2l}\right]L_{2n+2}(u)$$
$$+(u^2+4)\left[\sum_{l=0}^{m}\binom{2m+1}{2l+1}(2n+2)^{2l+1}\right]F_{2n+2}(u)$$
$$-[(2n+1)^{2m+1}]L_{2n+1}(u).$$

H-576 ***Proposed by Paul S. Bruckman, Sacramento, CA***

Define the following constant,

$$C_2\equiv\prod_{p>2}\{1-1/(p-1)^2\}$$

as an infinite product over all odd primes p.

(A) Show that

$$C_2=\sum_{n=1}^{\infty}\mu(2n-1)/\{\phi(2n-1)\}^2,$$

where $\mu(n)$ and $\phi(n)$ are the Möbius and Euler functions, respectively.

(B) Let $R(n)=\sum_{d\mid n}\mu(n/d)2^d$. Show that

$$C_2=\prod_{n=2}^{\infty}\{\zeta^*(n)\}^{-R(n)/n},$$

where $\zeta(n)=\sum_{k=1}^{\infty}k^{-n}$ is the Riemann zeta function (with $n>1$), and

$$\zeta^*(n)=\sum_{k=1}^{\infty}(2k-1)^{-n}=(1-2^{-n})\zeta(n).$$

Note: C_2 is the "twin-primes" constant that enters into Hardy and Littlewood's "extended" conjectures regarding the distribution of twin primes and Goldbach's conjecture.

SOLUTIONS

Comment by H.-J. Seiffert

In my solution to Prob. H-562, I gave a valid proof for the identity

$$L_{2n+1}=4^n-5\sum_{k=0}^{[\frac{n-2}{5}]}\binom{2n+1}{n-5k-2},$$

n a nonnegative integer, as stated in the original proposal. Therefore, the word "corrected" is meaningless.

Symmetry

H-564 ***Proposed by Stanley Rabinowitz, Westford, MA***
(Vol. 38, no. 4, August 2000)

Let k be a positive integer and let $a_0 = 1$. Find integers $a_1, a_2, \ldots, a_k$ and $b_0, b_1, b_2, \ldots, b_k$ such that

$$\sum_{i=0}^{k} a_i L_{n+i}^{2k} = \sum_{i=0}^{k} b_i F_{n+i}^{2k}$$

is true for all integers n. Prove that your answer is unique.

For example, when $k = 4$, we have the identity

$$L_n^8 + 21L_{n+1}^8 + 56L_{n+2}^8 + 21L_{n+3}^8 + L_{n+4}^8 = 625(F_n^8 + 21F_{n+1}^8 + 56F_{n+2}^8 + 21F_{n+3}^8 + F_{n+4}^8).$$

Solution by L. A. G. Dresel, Reading, England

Symmetry and Uniqueness

Let Σ_i denote the sum from $i = 0$ to k.

Let $I_k(\langle a_i, b_i\rangle, n)$ denote $\Sigma_i\{a_i(L_{n+i})^{2k} - b_i(F_{n+i})^{2k}\}$. Then, if $I_k(\langle a_i, b_i\rangle, n) = 0$ for all n and we put $n = -m-k$, we have $\Sigma_i\{a_i(L_{i-m-k})^{2k} - b_i(F_{i-m-k})^{2k}\} = 0$, and because $(L_{-t})^2 = (L_t)^2$ and $(F_{-t})^2 = (F_t)^2$ for all t, we obtain $\Sigma_i\{a_i(L_{m+k-i})^{2k} - b_i(F_{m+k-i})^{2k}\} = 0$ for all m. Finally, putting $j = k - i$, we obtain $\Sigma_j\{a_{k-j}(L_{m+j})^{2k} - b_{k-j}(F_{m+j})^{2k}\} = 0$. This shows that, if $I_k(\langle a_i, b_i\rangle, n) = 0$ for all n, then we also have $I_k(\langle a_{k-i}, b_{k-i}\rangle, m) = 0$ for all m. For this to represent the same identity, we must have $a_{k-i} = \lambda a_i$,, $b_{k-i} = \lambda b_i$, which leads to $\lambda^2 = 1$. This gives either the "symmetric" solution, $a_{k-i} = a_i$, $b_{k-i} = b_i$, or the "anti-symmetric" solution $a_{k-i} = -a_i$, $b_{k-i} = -b_i$. In general, if, say, the identity $I_k(\langle p_i, q_i\rangle, n) = 0$ is true, we also have $I_k(\langle p_{k-i}, q_{k-i}\rangle, n) = 0$, and since these identities are linear and homogeneous in the coefficients, it follows that we also have the "symmetric" identity $I_k(\langle p_i + p_{k-i}, q_i + q_{k-i}\rangle, n) = 0$ and the "anti-symmetric" identity $I_k(\langle p_i - p_{k-i}, q_i - q_{k-i}\rangle, n) = 0$. Therefore, we can prove the uniqueness of our solution for a particular k if we can show that there is only one symmetric solution and no anti-symmetric solution.

We shall also use the notation $A_i = a_i + 5^{-k}b_i$ and $D_i = a_i - 5^{-k}b_i$, and we note that A_i and D_i then have the same symmetry property as a_i.

<u>For $k = 1$</u>, the simplest case, we have,

$$(L_n)^2 = (\alpha^n + \beta^n)^2 = L_{2n} + 2(-1)^n \quad \text{and} \quad 5(F_n)^2 = (\alpha^n - \beta^n)^2 = L_{2n} - 2(-1)^n,$$

so that

$$\Sigma_i\{a_i(L_{n+i})^2 - b_i(F_{n+i})^2\} = D_0L_{2n} + D_1L_{2(n+1)} + 2(-1)^n\{A_0 - A_1\} = 0.$$

Since this is to be true for all n, we must have $D_0 = D_1 = 0$ and $A_0 = A_1$, so that we have the unique solution

$$(L_n)^2 + (L_{n+1})^2 = 5\{(F_n)^2 + (F_{n+1})^2\}.$$

The Reduction Algorithm for Even k

By rearranging the terms in the binomial expansion $(L_n)^{2k} = (\alpha^n + \beta^n)^{2k}$, or using equations (79) and (81) of [1], when k is even, say $k = 2h$, we obtain

$$(L_n)^{2k} = c_0 L_{2nk} + c_1(-1)^n L_{2n(k-1)} + c_2 L_{2n(k-2)} + \cdots + c_{k-1}(-1)^n L_{2n} + c_k$$

and

$$5^k (F_n)^{2k} = c_0 L_{2nk} - c_1(-1)^n L_{2n(k-1)} + c_2 L_{2n(k-2)} - \cdots - c_{k-1}(-1)^n L_{2n} + c_k,$$

where $c_0, c_1, \ldots, c_k$ are the first $k+1$ coefficients in the binomial expansion $(1+Z)^{2k}$.

We note that the right sides of these formulas differ only in the signs of the terms involving the factor $(-1)^n$. Therefore, if $A = a + 5^{-k}b$ and $D = a - 5^{-k}b$, we have

$$\{a(L_n)^{2k} - b(F_n)^{2k}\} = D\{c_0 L_{2kn} + c_2 L_{2(k-2)n} + \cdots + c_k\} + (-1)^n A\{c_1 L_{2(k-1)n} + \cdots + c_{k-1} L_{2n}\}$$

for all n. For large n, each of the $k+1$ terms on the right is of a different order of magnitude, so that to satisfy the identity $\Sigma_i \{a_i(L_{n+i})^{2k} - b_i(F_{n+i})^{2k}\} = 0$ we must satisfy the $k+1$ equations $\Sigma_i D_i L_{2s(n+i)} = 0$ (for $s = k, k-2, \ldots, 2$), $\Sigma_i D_i = 0$, and $\Sigma_i (-1)^i A_i L_{2s(n+i)} = 0$ (for $s = k-1, k-3, \ldots, 1$). Letting $y = L_{2s}$, we have the recurrence given by (17a) of [1], namely,

$$L_{2s(n+i)} - yL_{2s(n+i+1)} + L_{2s(n+i+2)} = 0.$$

We can use this to eliminate the first and last terms in each of the summations involving $L_{2s(n+i)}$, and then repeat this process until we are left with only the middle three terms. For simplicity, we put $C_i = (-1)^i A_i$, and note that C_i has the same symmetry as A_i because, when k is even, $(-1)^{k-i} = (-1)^i$. Now consider the reduction of $\Sigma_i C_i L_{2s(n+i)} = 0$, first to

$$(yC_0 + C_1)L_{2s(n+1)} + (C_2 - C_0)L_{2s(n+2)} + C_3 L_{2s(n+3)} + \cdots = 0,$$

and more generally to

$$R_i L_{2s(n+i)} + (C_{i+1} - R_{i-1})L_{2s(n+i+1)} + C_{i+2} L_{2s(n+i+2)} + \cdots = 0$$

for $i = 1, 2, \ldots, h-1$, where $R_0 = C_0$, $R_{-1} = 0$, and $R_i = yR_{i-1} - R_{i-2} + C_i$ for $i \geq 1$. Thus, R_i is a polynomial in y of degree i. For a symmetric solution, the final reduction gives

$$R_{h-1} L_{2s(n+h-1)} + (C_h - 2R_{h-2})L_{2s(n+h)} + R_{h-1} L_{2s(n+h+1)} = 0,$$

where the middle term includes a contribution of R_{h-2} from both the left and the right. This final reduction must be a multiple of the recurrence relation, so that we have $yR_{h-1} + C_h - 2R_{h-2} = 0$, giving $R_h - R_{h-2} = 0$, an equation of degree h which must have the h roots $y = L_{2s}$ for $s = 1, 3, \ldots, k-1$. This determines the $h+1$ coefficients C_0 to C_h, except for an arbitrary factor, while the remaining coefficients are determined by the symmetry condition $C_{k-i} = C_i$.

If, on the other hand, the C_i were anti-symmetric, our reduction would lead to

$$R_{h-1} L_{2s(n+h-1)} + C_h L_{2s(n+h)} - R_{h-1} L_{2s(n+h+1)} = 0$$

with $C_h = 0$, giving $R_{h-1} = 0$. This is an equation of degree $h-1$ in y, which cannot have h roots unless $C_i = 0$ for all i, giving the only anti-symmetric solution for C_i. Similarly, the only anti-symmetric solution for D_i is $D_i = 0$ for all i.

Finally, we have to look for *symmetric* solutions for the coefficients D_i. But then we cannot satisfy the additional equation $\Sigma_i D_i = 0$, unless $D_i = 0$ for all i. Hence, we have $b_i = 5^k a_i$, so that our symmetric solution for $C_i = (-1)^i A_i = 2(-1)^i a_i$, as determined by the equation $R_h - R_{h-2} = 0$, gives the unique solution for the coefficients a_i.

Defining $P_i = R_i - R_{i-2}$ and using the recurrence $R_i = yR_{i-1} - R_{i-2} + C_i$, we obtain a recurrence for P_i, namely, $P_i = yP_{i-1} - P_{i-2} + (C_i - C_{i-2})$, a polynomial of degree i. Hence, we have $P_0 = C_0$, $P_1 = yC_0 + C_1$, $P_2 = y(yC_0 + C_1) + (C_2 - 2C_0)$, $P_3 = y^2(yC_0 + C_1) + y(C_2 - 3C_0) + (C_3 - 2C_1)$, and $P_4 = y^3(yC_0 + C_1) + y^2(C_2 - 4C_0) + y(C_3 - 3C_1) + (C_4 - 2C_2 + 2C_0)$.

We now consider the equation $P_h = 0$, identifying its roots for $k = 2h = 2, 4, 6$, and 8.

For $k = 2$, we have $P_1 = yC_0 + C_1 = 0$, giving $yA_0 - A_1 = 0$ for $y = L_2 = 3$. Thus, $\langle a_i \rangle = \langle 1, 3, 1 \rangle$, so that $(L_n)^4 + 3(L_{n+1})^4 + (L_{n+2})^4 = 25\{(F_n)^4 + 3(F_{n+1})^4 + (F_{n+2})^4\}$.

For $k = 4$, we have $P_2 = y(yC_0 + C_1) + (C_2 - 2C_0) = 0$, giving $y(yA_0 - A_1) + (A_2 - 2A_0) = 0$ for $y = L_2$ and L_6. Hence, taking $a_0 = 1$, we obtain $a_1 = L_2 + L_6 = 3 + 18 = 21$ and $a_2 = L_2 L_6 = 54$, giving $\langle a_i \rangle = \langle 1, 21, 56, 21, 1 \rangle$ and $b_i = 5^4 a_i$, which agrees with the solution given by the proposer.

For $k = 6$, we have $P_3 = y^2(C_0 y + C_1) + y(C_2 - 3C_0) + (C_3 - 2C_1) = 0$ for $y = L_2$, L_6, L_{10}. This leads to $a_0 = 1$, $a_1 = L_2 + L_6 + L_{10} = 21 + 123 = 144$, $a_2 - 3 = L_2 L_6 + (L_2 + L_6) L_{10} = 54 + 21 \cdot 123 = 2637$, $a_3 - 2a_1 = 54 \cdot 123 = 6642$. Therefore, we obtain $\langle a_1 \rangle = \langle 1, 144, 2640, 6930, 2640, 144, 1 \rangle$ and $b_i = 5^6 a_i$.

For $k = 8$, we have $y^3(C_0 y + C_1) + y^2(C_2 - 4C_0) + y(C_3 - 3C_1) + (C_4 - 2C_2 + 2C_0) = 0$ for $y = L_2$, L_6, L_{10}, L_{14}, leading to $a_0 = 1$, $a_1 = 144 + 843 = 987$, $a_2 - 4 = 2637 + 144 \cdot 843 = 124029$, $a_3 - 3a_1 = 6642 + 2637 \cdot 843 = 2229633$, $a_4 - 2a_2 + 2 = 6642 \cdot 843 = 5599206$. Thus, we obtain $b_i = 5^8 a_i$, with $\langle a_i \rangle = \langle 1, 987, 124033, 2232594, 5847270, 2232594, 124033, 987, 1 \rangle$.

The Reduction Algorithm for Odd k

Proceeding as before, when $k = 2h + 1$ we have

$$(L_n)^{2k} = c_0 L_{2nk} + c_1(-1)^n L_{2n(k-1)} + c_2 L_{2n(k-2)} + \cdots + c_{k-1} L_{2n} + (-1)^n c_k,$$

and

$$5^k (F_n)^{2k} = c_0 L_{2nk} - c_1(-1)^n L_{2n(k-1)} + c_2 L_{2n(k-2)} - \cdots + c_{k-1} L_{2n} - (-1)^n c_k.$$

Taking $A = a + 5^{-k} b$ and $D = a - 5^{-k} b$ as before, we now have

$$\{a(L_n)^{2k} - b(F_n)^{2k}\} = D\{c_0 L_{2kn} + c_2 L_{2(k-2)n} + \cdots + c_{k-1} L_{2n}\} + (-1)^n A\{c_1 L_{2(k-1)n} + \cdots + c_k\}.$$

To satisfy the identity $\Sigma_i \{a_i (L_{n+i})^{2k} - b_i (F_{n+i})^{2k}\} = 0$ we have the $k+1$ equations $\Sigma_i D_i L_{2s(n+i)} = 0$ (for $s = k$, $k-2, \ldots, 1$), $\Sigma_i (-1)^i A_i L_{2s(n+i)} = 0$ (for $s = k-1$, $k-3, \ldots, 2$) and $\Sigma_i (-1)^i A_i = 0$. The last equation will require A_i to be symmetric, and therefore $C_i = (-1)^i A_i$ will be anti-symmetric, as k is odd. The formulas for R_i in the reduction algorithm are the same as before, and at the penultimate stage we have four middle terms remaining, namely,

$$R_{h-1} L_{2s(n+h-1)} + (C_h - R_{h-2}) L_{2s(n+h)} - (C_h - R_{h-2}) L_{2s(n+h+1)} - R_{h-1} L_{2s(n+h+2)} = 0.$$

This finally reduces to $(R_h + R_{h-1}) L_{2s(n+h)} - (R_h + R_{h-1}) L_{2s(n+h+1)} = 0$ for all n, giving $R_h + R_{h-1} = 0$, an equation of degree h with the h roots $y = L_{2s}$, for $s = 2, 4, \ldots, k-1$. This determines C_0 to C_h, and hence the symmetric $A_i = (-1)^i C_i$.

Turning to the equations for D_i, they also reduce to an equation of degree h in y with roots $y = L_{2s}$, but now we have the $h+1$ values $s = 1, 3, \ldots, k$. Hence, $D_i = 0$ for all i, and we have $b_i = 5^k a_i$, and our symmetric solution for $A_i = (-1)^i C_i$. gives the unique solution for the coefficients a_i and b_i.

Defining $Q_i = R_i + R_{i-1}$ and using the recurrence $R_i = yR_{i-1} - R_{i-2} + C_i$, we obtain $Q_i = yQ_{i-1} - Q_{i-2} + (C_1 + C_{i-1})$, a polynomial in y of degree i. Hence, we have $Q_0 = C_0$, $Q_1 = (y+1)C_0 + C_1$, $Q_2 = y(yC_0 + C_0 + C_1) + (C_2 + C_1 - C_0)$, $Q_3 = y^2(yC_0 + C_0 + C_1) + y(C_2 + C_1 - 2C_0) + (C_3 + C_2 - C_1 - C_0)$.

We now consider the equation $Q_h = 0$, identifying its roots for $k = 2h+1 = 3, 5, 7$.

For $k = 3$, $Q_1 = (y+1)C_0 + C_1 = (y+1)A_0 - A_1 = 0$ for $y = L_4 = 7$, giving $A_1 = 8A_0$ and, finally, $\langle a_i \rangle = \langle 1, 8, 8, 1 \rangle$ and $b_i = 5^3 a_i$.

For $k = 5$, $Q_2 = y(yC_0 + C_0 + C_1) + (C_2 + C_1 - C_0) = 0$, thus $y(yA_0 + A_0 - A_1) + (A_2 - A_1 - A_0) = 0$ for $y = L_8 = 47$ and $y = L_4$. Therefore, $(A_1 - A_0) = (47+7)A_0$ and $(A_2 - A_1 - A_0) = 7 \cdot 47 A_0$, so we have the solution $\langle a_i \rangle = \langle 1, 55, 385, 385, 55, 1 \rangle$ and $b_i = 5^5 a_i$.

For $k = 7$, $Q_3 = y^2(yC_0 + C_0 + C_1) + y(C_2 + C_1 - 2C_0) + (C_3 + C_2 - C_1 - C_0) = 0$ for $y = 7, 47$, and $y = L_{12} = 322$. Hence, $(A_1 - A_0) = (54 + 322)A_0$, $(A_2 - A_1 - 2A_0) = (7 \cdot 47 + 54 \cdot 322)A_0 = 17717A_0$, and $(A_3 - A_2 - A_1 + A_0) = 7 \cdot 47 \cdot 322 A_0 = 105938A_0$, so we have the solution $\langle a_i \rangle = \langle 1, 377, 18096, 124410, 124410, 18096, 377, 1 \rangle$ and $b_i = 5^7 a_i$.

Concluding Remarks

It is clear that, using these methods, we can obtain a unique solution for the coefficients for any value of k, and that $a_{k-i} = a_i$ and $b_i = 5^k a_i$.

We also have $a_1 = F_{2k}$ for all k. When k is odd, we have $a_1(k) = 1 + L_4 + \cdots + L_{2k-2}$, while for k even we have $a_1(k) = L_2 + L_6 + \cdots + L_{2k-2}$. Therefore, $a_1(k) - a_1(k-2) = L_{2k-2}$ for all k. But

$$L_{2k-2} = F_{2k-1} + F_{2k-3} = F_{2k} - F_{2k-2} + (F_{2k-2} - F_{2k-4}) = F_{2k} - F_{2(k-2)},$$

and we have $a_1(2) = 3 = F_4$ and $a_1(3) = 8 = F_6$. Hence, by induction, $a_1(k) = F_{2k}$ for all k.

In the numerical results for even $k = 2h = 4, 6, 8$, we note that the coefficients a_2 to a_h are all divisible by F_{2k-2}, whereas for odd $k = 2h+1 = 5, 7$ we find a_1 to a_h are divisible by F_{2k}. We conjecture that these results are true for all k, but time does not permit us to pursue this further here.

Reference

1. S. Vajda. *Fibonacci & Lucas Numbers, and the Golden Section*. Chichester: Ellis Horwood Ltd., 1989.

Also solved by P. Bruckman, H.-J. Seiffert, and the proposer.

A Prime Example

H-565 ***Proposed by Paul S. Bruckman, Berkeley, CA***
(Vol. 38, no. 4, August 2000)

Let p be a prime with $p \equiv -1 \pmod{2m}$, where $m \geq 3$ is an odd integer. Prove that all residues are m^{th} powers (mod p).

Solution by the proposer

Given any residue $x \pmod p$, let $y \equiv (x/p)x^{(p+1)/2m} \pmod p$. Clearly, y is a well-defined residue $\pmod p$. We make use of the well-known result: $(x/p) \equiv x^{(p-1)/2} \pmod p$. Then

$$y^m \equiv (x/p)^m x^{(p+1)/2} \equiv (x/p)^m x^{(p-1)/2} x \equiv (x/p)^{(m+1)} x \equiv x \pmod p,$$

since m is odd. We then see that x [an arbitrary residue $\pmod p$] is an m^{th} power $\pmod p$. Q.E.D.

Also solved by L. A. G. Dresel, R. Martin, and H.-J. Seiffert.

Late Acknowledgment: H.-J. Seiffert solved H-563.

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The Fibonacci Association

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The purpose of the conference is to bring together people from all branches of mathematics and science who are interested in Fibonacci numbers, their applications and generalizations, and other special number sequences. For the conference *Proceedings*, manuscripts that include new, unpublished results (or new proofs of known theorems) will be considered. A manuscript should contain an abstract on a separate page. For papers not intended for the *Proceedings*, authors may submit just an abstract, describing new work, published work or work in progress. Papers and abstracts, which should be submitted in duplicate to F.T. Howard at the address below, are due by May 1, 2002. Authors of accepted submissions will be allotted twenty minutes on the conference program. Questions about the conference may be directed to:

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