Structure-Aware Private Set Intersection from Function Secret Sharing

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Joint work with Benjamin Goff, Peihan Miao, Mike Rosulek and Jaspal Singh

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Private Set Intersection (PSI)

Alice

input S_A

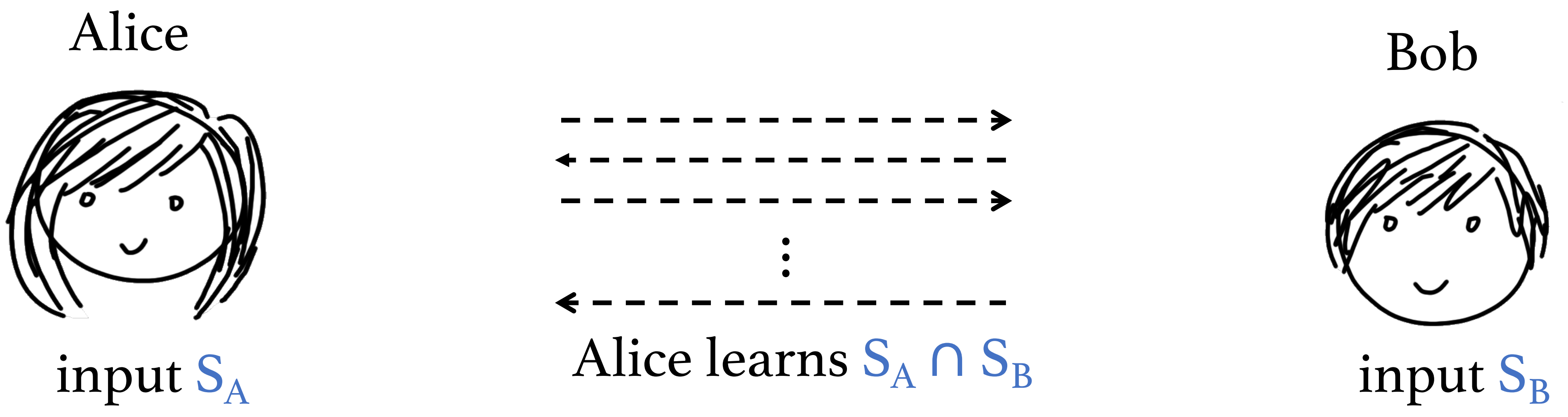
{p, r, i, v, a, t, e}

Bob

input S_B

{s, e, c, u, r, i, t, y}

Private Set Intersection (PSI)



Alice

input S_A

Alice learns $S_A \cap S_B$

Bob

input S_B

$\{p, \underline{r}, \underline{i}, v, a, \underline{t}, \underline{e}\}$

identifies only common elements

$\{?, e, ?, ?, r, i, t, ?\}$

learns nothing about Alice's input

PSI Research

Approaches

- Diffie-Hellman [Mea86, HFH99, JL10, DKT10, IKN+20, RT21…]
- Oblivious Polynomial Evaluation [FNP04, KS05, dMRY11…]
- RSA [DT10, ADT11]
- Bloom Filters [DCW13, RR17a]
- FHE [CLR17, CHLR18, CMDG+21]
- Circuit-based [HEK12, PSSZ15, PSWW18, PSTY19, GarimellaMR+21]
- OT [PSZ14, PSSZ15, KKRT16, RR17, PRTY19, CM20, PRTY20, RS21, GarimellaPR+21]
- Vector OLE [RS21, GarimellaPR+21, CRR21, RR22, BPSY23…]

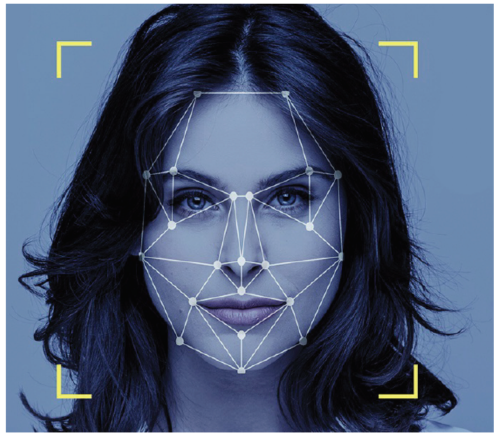
Settings

- Semi-honest/Malicious [RR17, OOS17, CHLR18, PRTY20, RS21, GarimellaPR$^+$21, BPSY23…]
- Plain/Cardinality/Associated-sum: [PSTY19, KK20, MPR$^+$20, IKN$^+$20, GarimellaMR$^+$21, RS21, CGS22...]
- PS Union: [DC17, KRTY19, GarimellaPR+21, JSZ+22, LG23, BPSY23, GNT24…]
- Balanced/Unbalanced/Laconic: [ABD$^+$21, ALOS22, DKL$^+$23, GHMM24..]
- Two-party/Multi-party: [HV17, NTY21, BMRR21, CDG$^+$21, GarimellaPR$^+$21, ENOP22, BHV$^+$23, GTY24..]
- Updatable: [KLS+17, ATD20, BMX22..]
- Fuzzy PSI: [CFR$^+$21,UCK$^+$21 ..]

Fuzzy matching

PSI enables exact matches between elements, but what if..

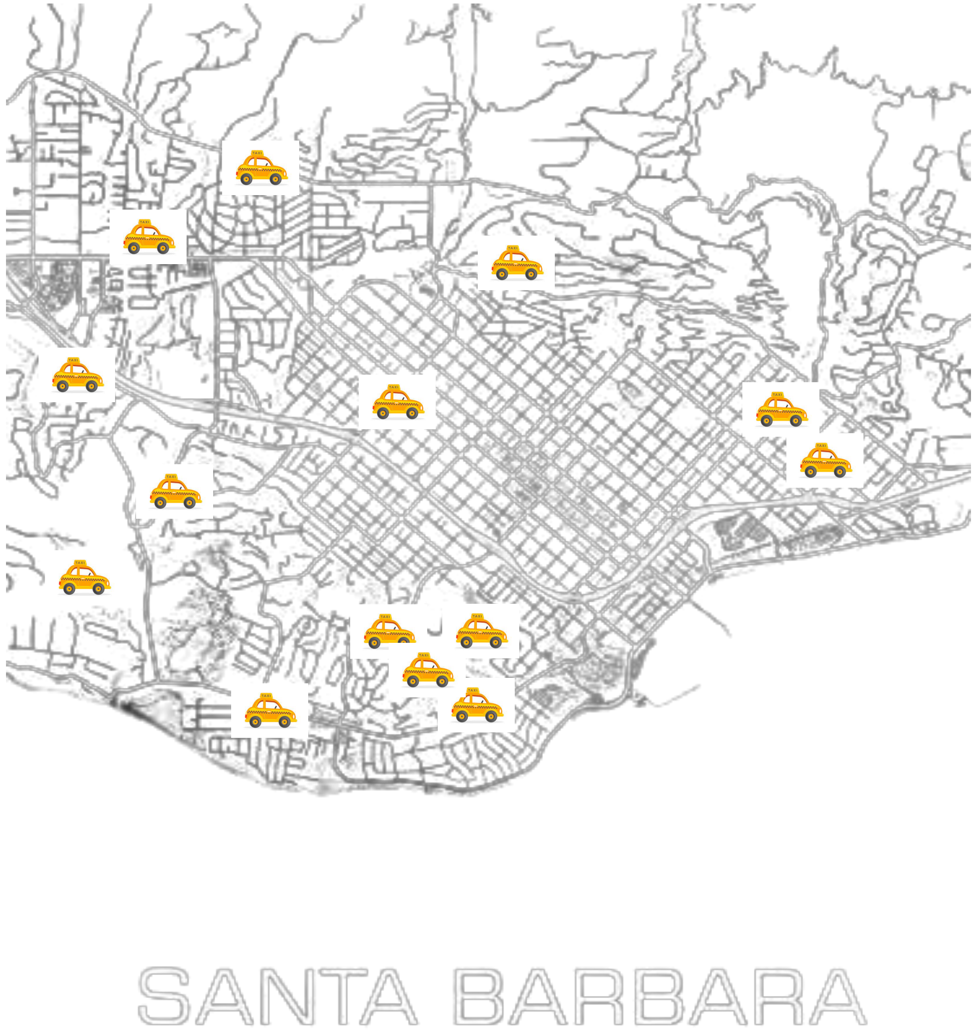
- Password breach with typos
- Biometrics fingerprint, facial recognition matching…
- Fuzzy Personal Identifiable Information (PII) matching – (name, physical address, contact, ..)
- Privacy-preserving ride-sharing application using fuzzy GPS matching (next slide)



Fuzzy matching

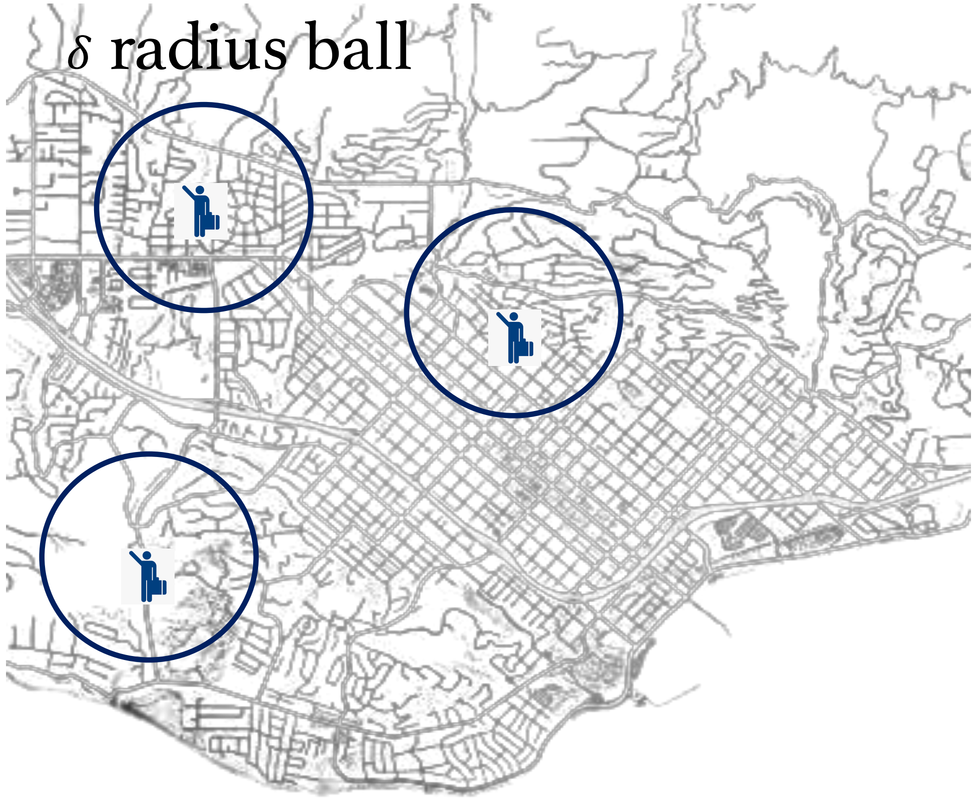
privacy-preserving ride hailing service

SANTA BARBARA

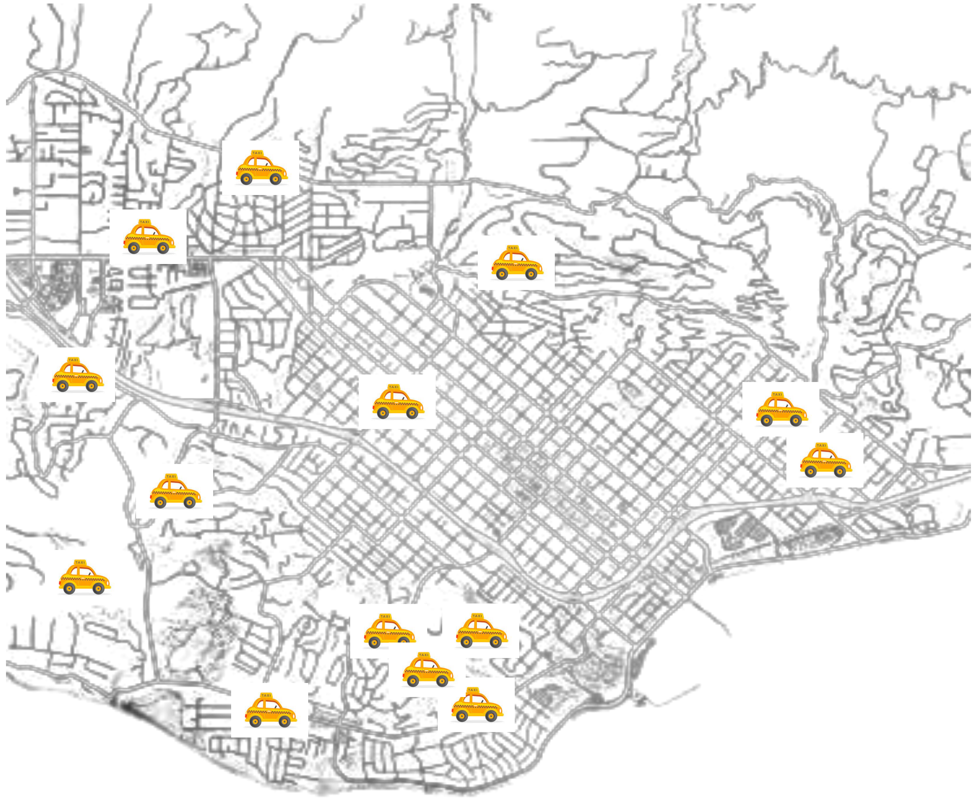


Fuzzy matching

privacy-preserving ride hailing service



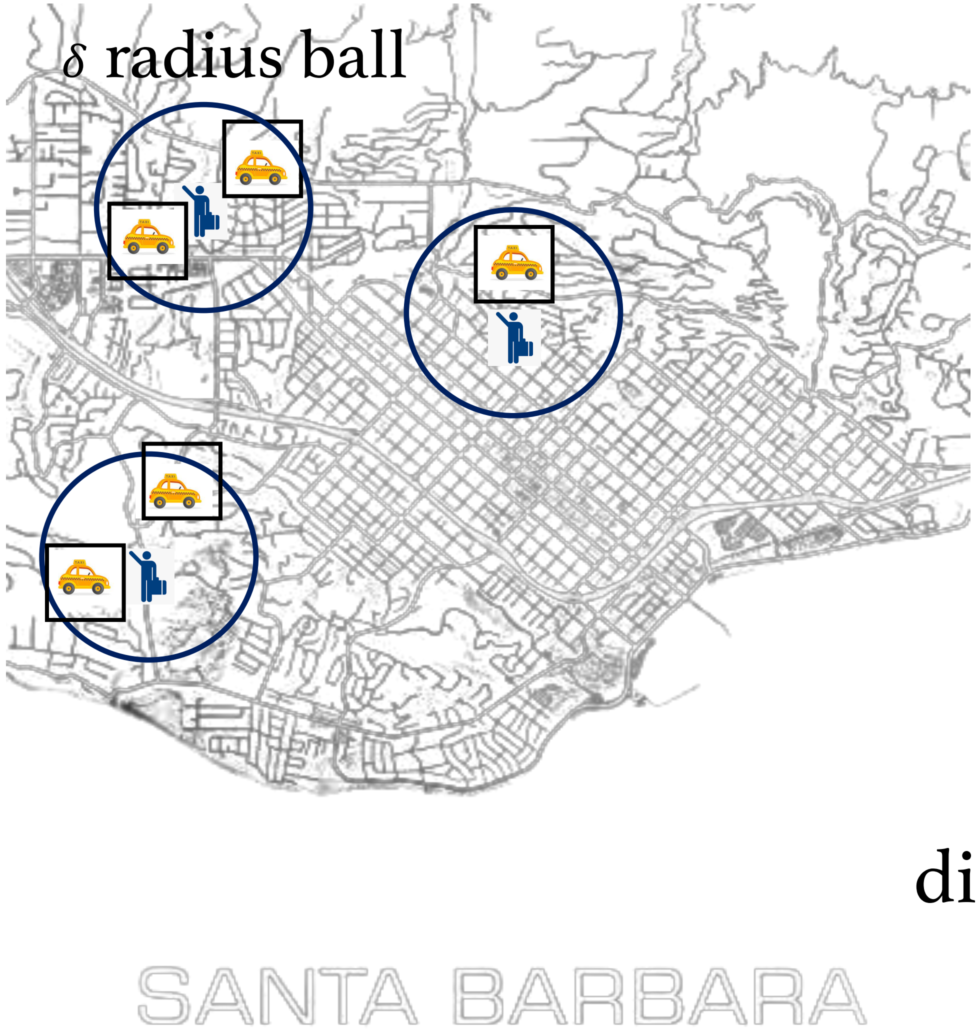
δ radius ball

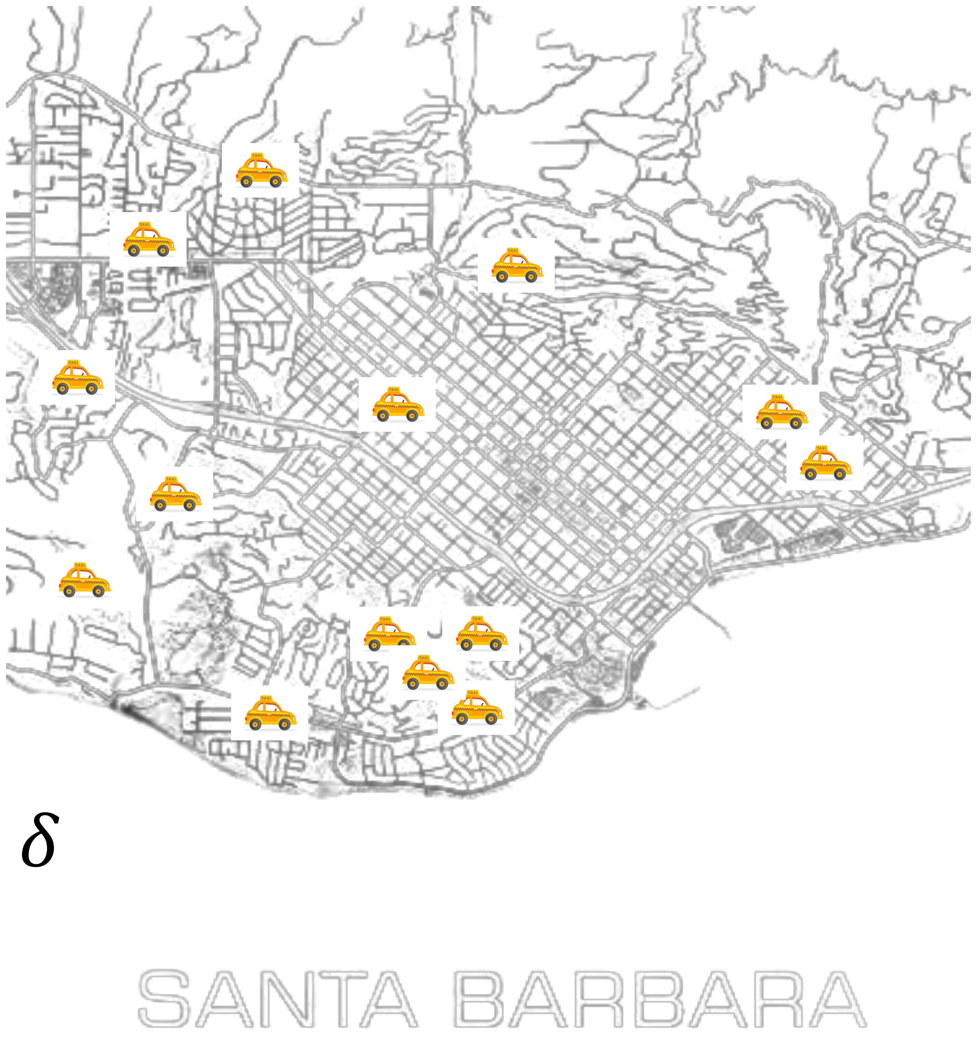


dist(🙋, 🚕) $\leq \delta$

Fuzzy matching

privacy-preserving ride hailing service





dist(🙋, 🚕) $\leq \delta$

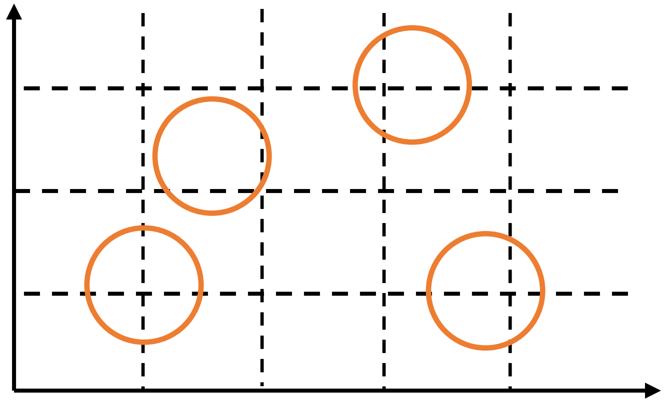
Naïve solution

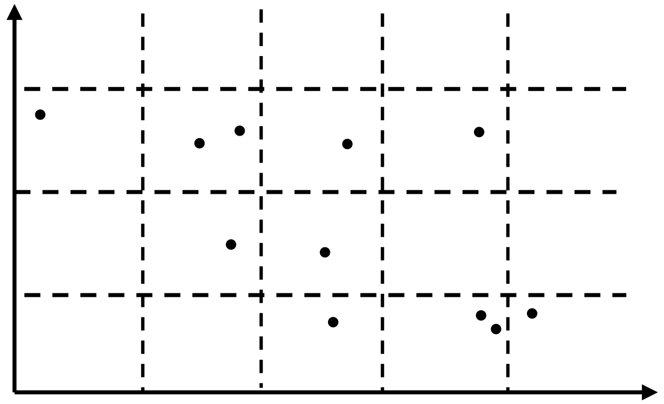
Alice's enumerates her structured input S_A

reduces to standard PSI

input S_A = {all the points inside structure}

input S_B





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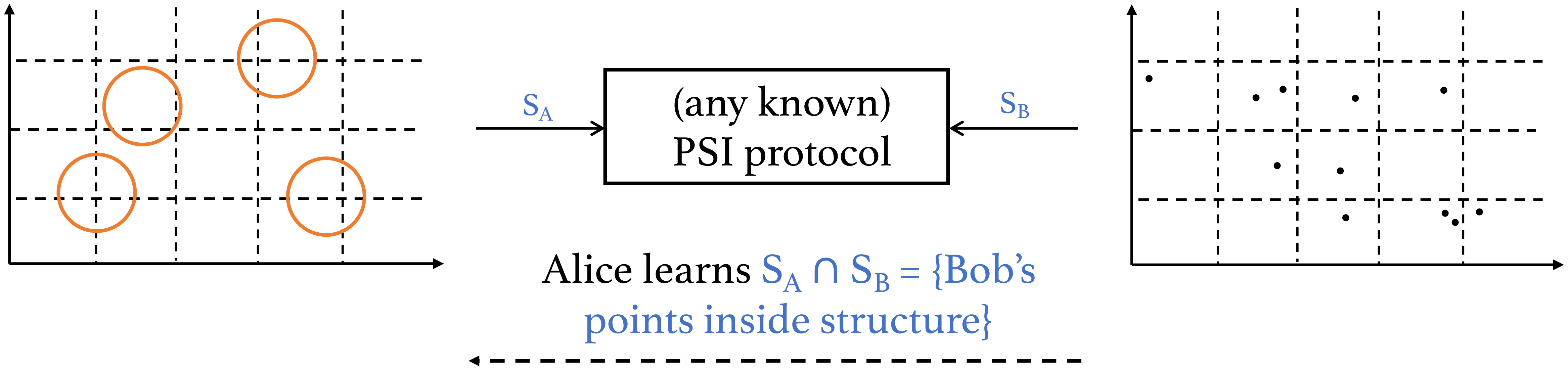


S_A

(any known)
PSI protocol

S_B

Alice learns $S_A \cap S_B$ = {Bob's points inside structure}

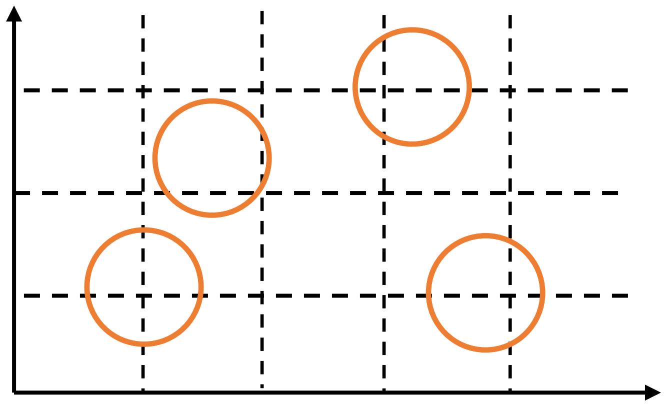
Naïve solution

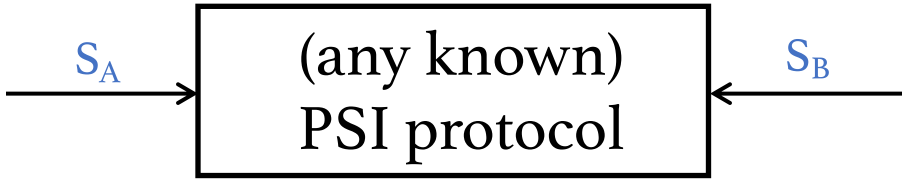
Comm and / or Comp cost $O((|S_A| + |S_B|).\kappa)$

~ total volume $|S_A|$ of balls in Alice's input

input S_A = {all the points inside structure}

input S_B

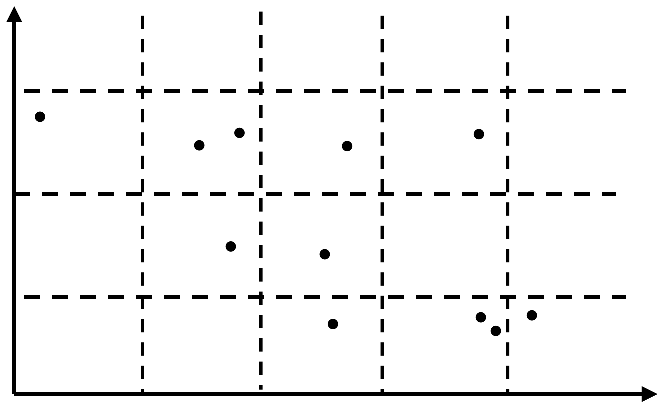




S_A

(any known) PSI protocol

S_B



Alice learns $S_A \cap S_B$ = {Bob's points inside structure}

Naïve solution

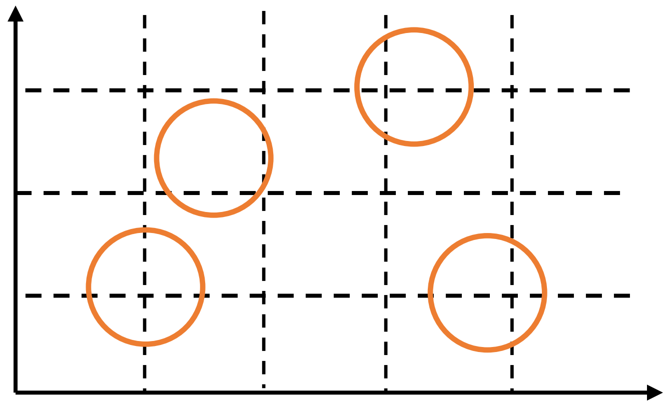
Can the protocol cost scale with description size (# of balls) in Alice's input?

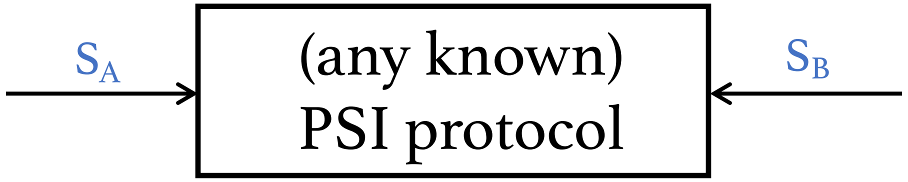
Comm and / or Comp cost $O((|S_A| + |S_B|). \kappa)$

~ total volume $|S_A|$ of balls in Alice's input

input S_A = {all the points inside structure}

input S_B

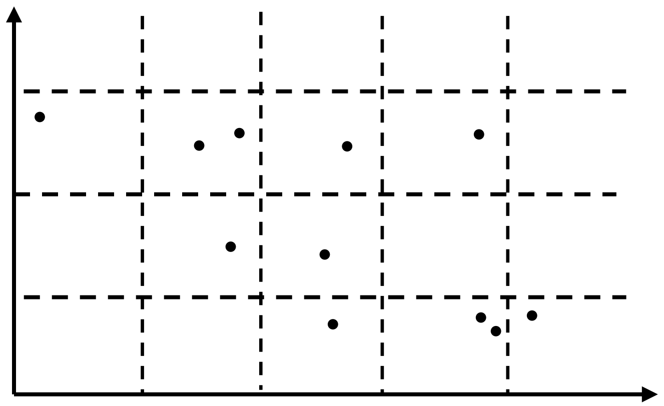




S_A

(any known)
PSI protocol

S_B



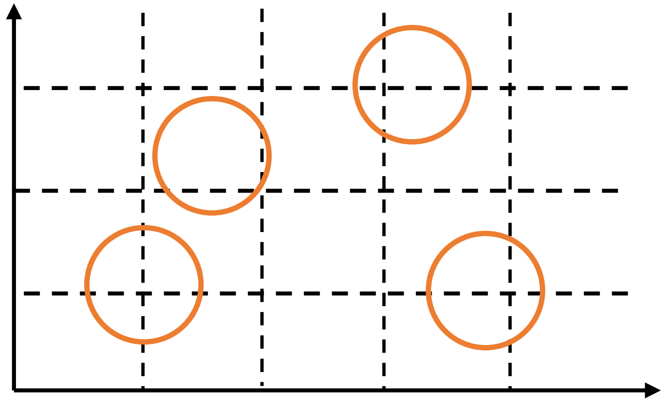
Alice learns $S_A \cap S_B$ = {Bob's points inside structure}

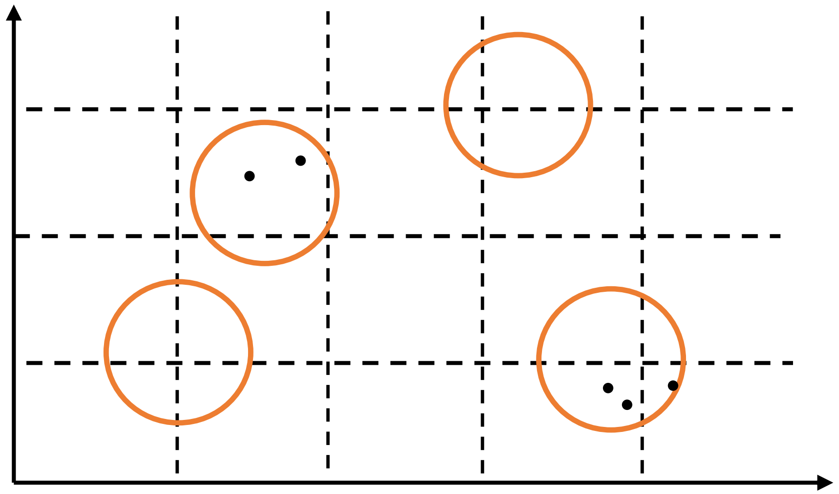
Structure-Aware Private Set Intersection (sa-PSI)

[GarimellaRosulekSingh22] a variant of PSI where Alice's input has a publicly known structure

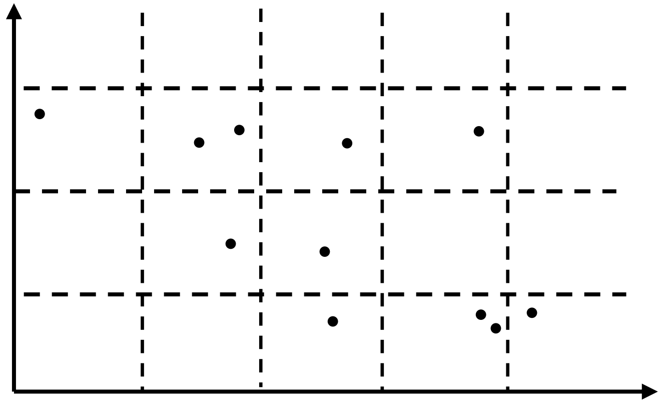
Examples - interval, ball or union of balls in some well-defined metric space, …

input S_A





input S_B



Alice learns $S_A \cap S_B$ = {all points inside balls}

Structure-Aware PSI, with applications to Fuzzy Matching. CRYPTO 2022. *Gayathri Garimella*, Mike Rosulek and Jaspal Singh

Research question:

Can we design “more efficient” PSI protocols, when one party has a structured input from a publicly known set family?

Research question:

Can we design "more efficient" PSI protocols, when one party has a structured input from a publicly known set family?

Can the communication and / or computation cost scales with a succinct description of the structured input, instead of the set cardinality?

might be prohibitively large for many realistic applications

Summary of results

Structure–Aware Private Set Intersection

Structure-Aware PSI, with applications to Fuzzy Matching. CRYPTO 2022.
Gayathri Garimella, Mike Rosulek and Jaspal Singh

- ✓ Semi-honest adversaries
- ✓ Comm ~ description size
- ✓ Tool: Boolean Function Secret sharing
- ✓ PSI construction for {union of L_∞ balls} set family

Summary of results

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Malicious-secure, Structure-Aware PSI.
CRYPTO 2023.
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Computation Efficient Structure Aware PSI.
CRYPTO 2024.
Gayathri Garimella, Benjamin Goff and Peihan Miao

- ✓ Semi-honest adversaries
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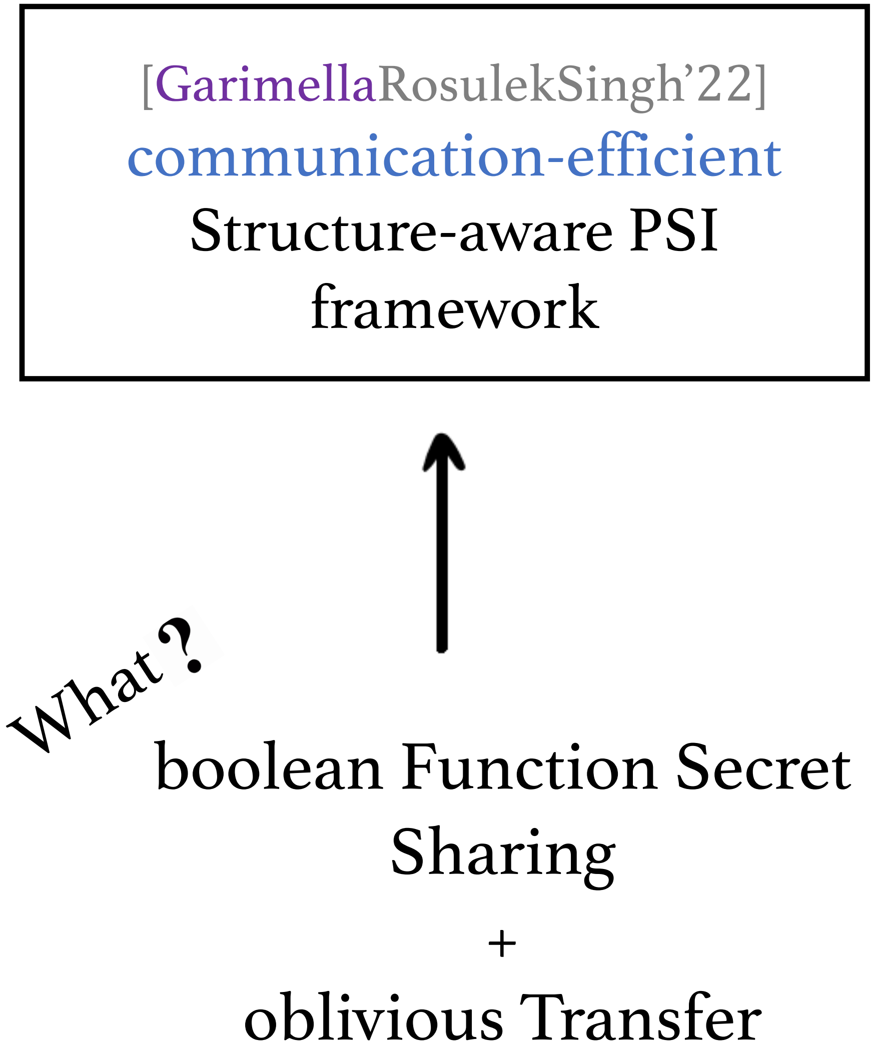
Result 1:



[GarimellaRosulekSingh'22]
communication-efficient
Structure-aware PSI
framework

What?

boolean Function Secret Sharing
+
oblivious Transfer

Building block 1:

Boolean Function Secret Sharing

given input $S_A \in S$ from a class of structured sets

boolean Function Secret Sharing (bFSS)

[BoyleGilboaIshai15] – style FSS for set membership in S_A function

Building block 1:

Boolean Function Secret Sharing

given input $S_A \in \mathcal{S}$ from a class of structured sets

<u>boolean Function Secret Sharing</u> (bFSS)

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share(S_A) ⟶ ░, ▤ where ░, ▤ $\approx \$\$$

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[BoyleGilboaIshai15] – style FSS for set membership in S_A function

share(S_A) $\longrightarrow$ ░ , ▤ where ░ , ▤ $\approx \$\$$

$\forall x \in S_A \Longrightarrow$ **ev**(░ , x) $\oplus$ **ev**(▤ , x) $=0$

$\forall x \notin S_A \Longrightarrow$ **ev**(░ , x) $\oplus$ **ev**(▤ , x) $=1$

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succinctness: | ▤ | , | ░ | $= \sigma <<$ | S_A |

Building block 1:

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share(S_A) ⟶ ⬚, ▤ where ⬚, ▤ ≈ $$

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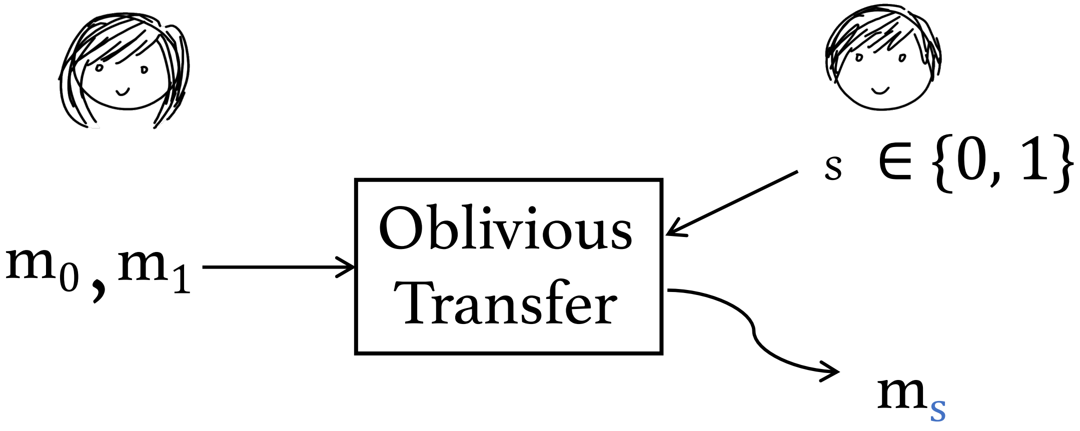
$\forall x \notin S_A \Longrightarrow$ **ev**(⬚ , x) $\oplus$ **ev**(▤ , x) = 1

succinctness: |▤|, |⬚| $= \sigma <<$ | S_A |

[BGI15, BGI16, BCG+21, BGIK22] - PRG based constructions for set families like {singleton, 1-d interval, d-dimensional interval..}

Building block 2:

Oblivious Transfer [Rabin'81]

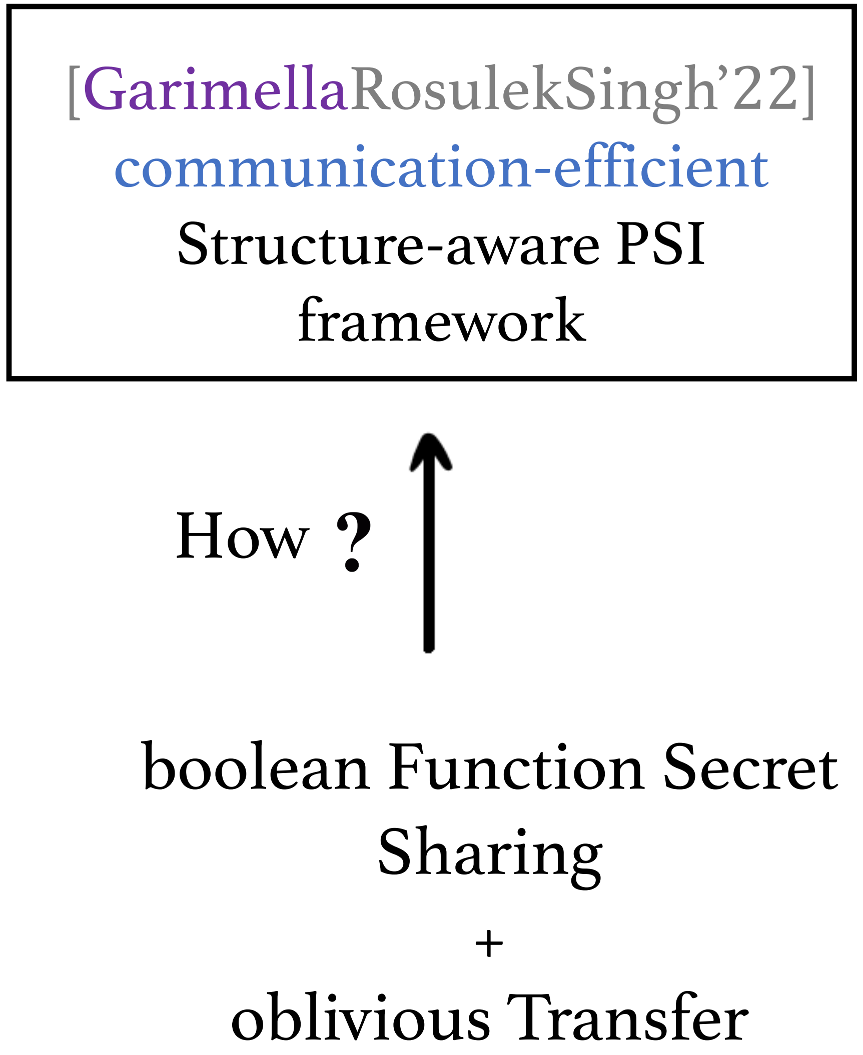


many OTs can be instantiated efficiently (largely using symmetric key operations) from OT extension [IKNP03]

How to exchange secrets with oblivious transfer. Cryptology ePrint Archive, 2005. Michael O Rabin.

Extending Oblivious Transfers Efficiently. CRYPTO 2003. Yuval Ishai, Joe Kilian, Kobbi Nissim and Erez Petrank.

Now, let's see how to realize sa-PSI



[GarimellaRosulekSingh'22]
communication-efficient
Structure-aware PSI
framework

How ?

boolean Function Secret
Sharing
+
oblivious Transfer

How does the sa-PSI protocol work?

assumptions: OT-hybrid, hamming correlation robust hash

input S_A

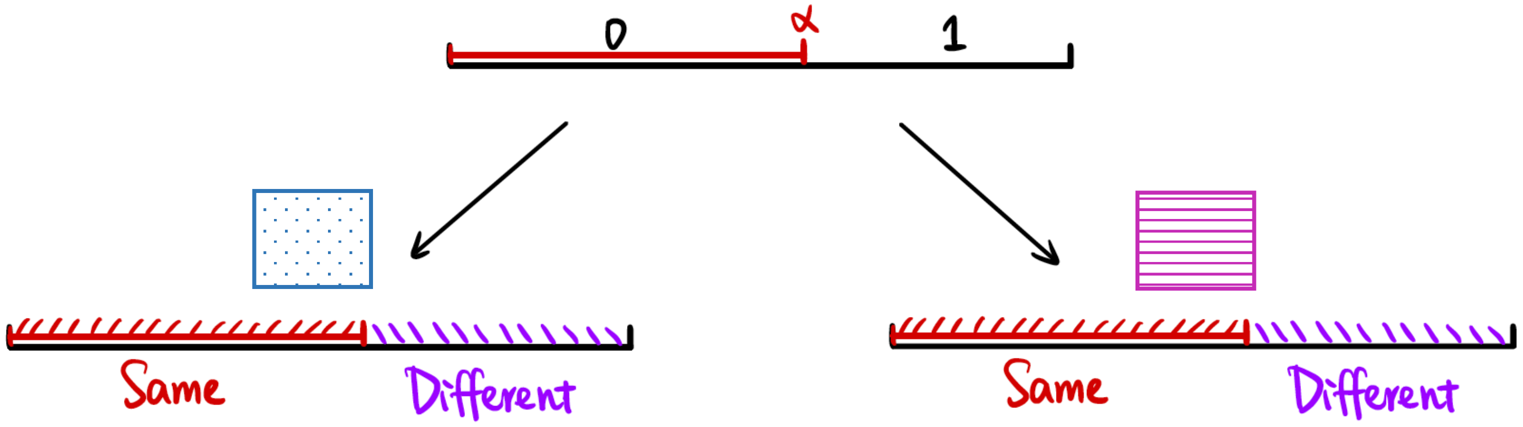
input S_B

1. Alice generates bFSS shares of her input S_A

Example: Alice's input is One-sided Interval

input S_A

1. Alice generates bFSS shares of her input S_A = {one-sided interval}



Recall definition:

given input A ∈ $\mathcal{S}$ from a class of structured sets

<u>boolean Function Secret Sharing</u> (bFSS)

[BoyleGilboaIshai15] – style FSS for set membership in A function

share(A) $\longrightarrow$ ▦, ▤ where ▦, ▤ $\approx \$\$$

$\forall x \in A \Longrightarrow$ ev(▦ , x) $\oplus$ ev(▤ , x) = 0

$\forall x \notin A \Longrightarrow$ ev(▦ , x) $\oplus$ ev(▤ , x) = 1

succinctness: |▤|, |▦| $= \sigma <<$ | S_A |

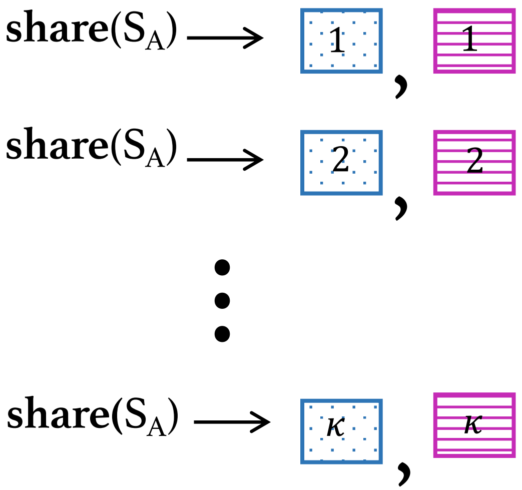
How does the sa-PSI protocol work?

input S_A

input S_B

1. generates κ instances of bFSS shares

2. picks κ choice bits to learn ░ or ▤



share(S_A) → ░1, ▤1

share(S_A) → ░2, ▤2

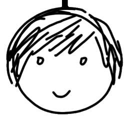
⋮

share(S_A) → ░κ, ▤κ

How does the sa-PSI protocol work?

input S_A

input S_B



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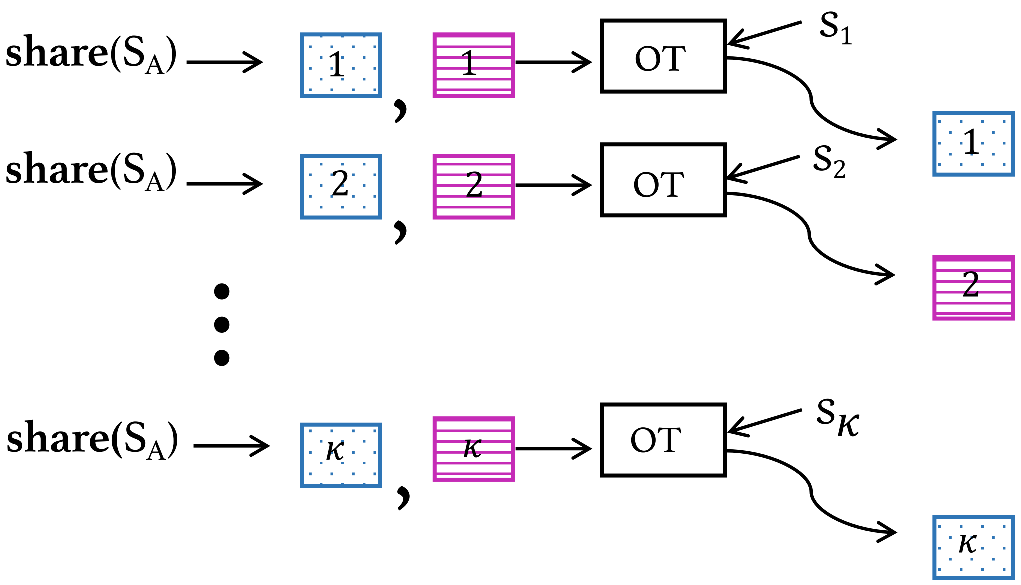

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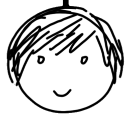

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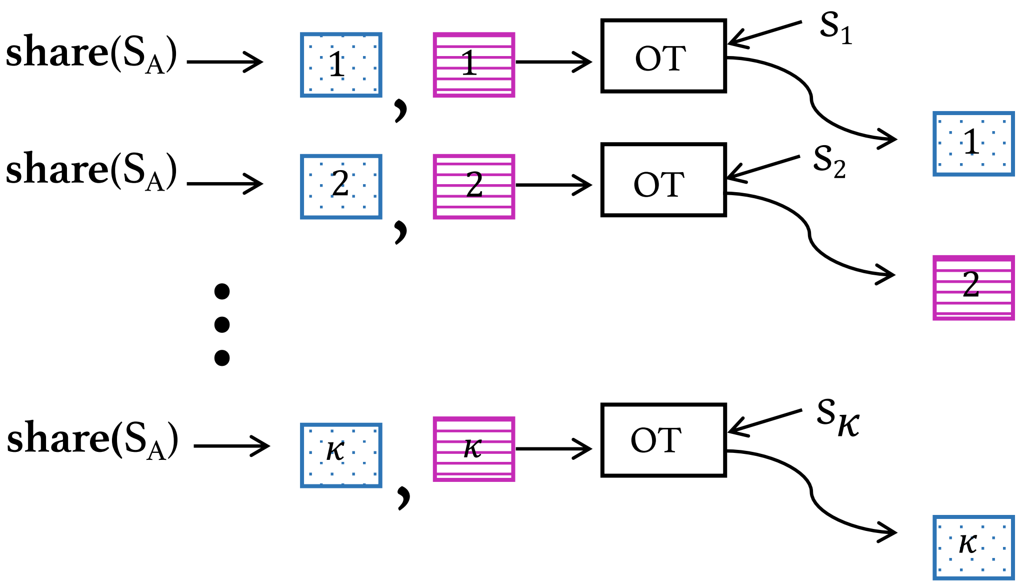
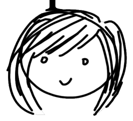


Bob computes F(x) on all his inputs

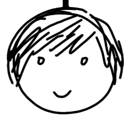
$$\boxed{\mathrm{F}(x) = \mathbf{H}(\mathbf{ev}(\boxed{1}, x) \| \mathbf{ev}(\boxed{2}, x) \| \cdots \mathbf{ev}(\boxed{\kappa}, x))}$$

How does the sa-PSI protocol work?

input S_A



input S_B



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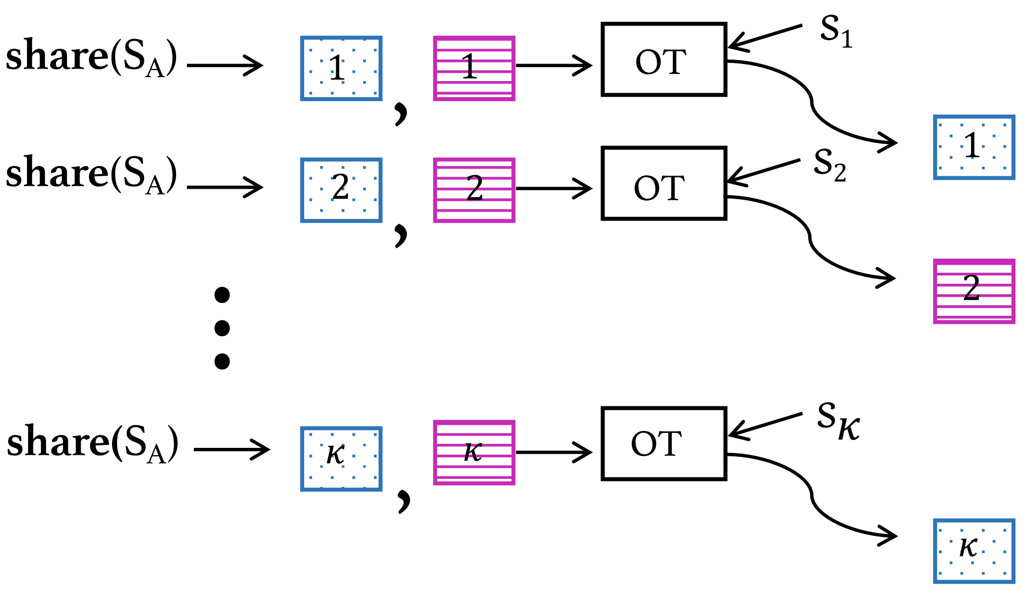
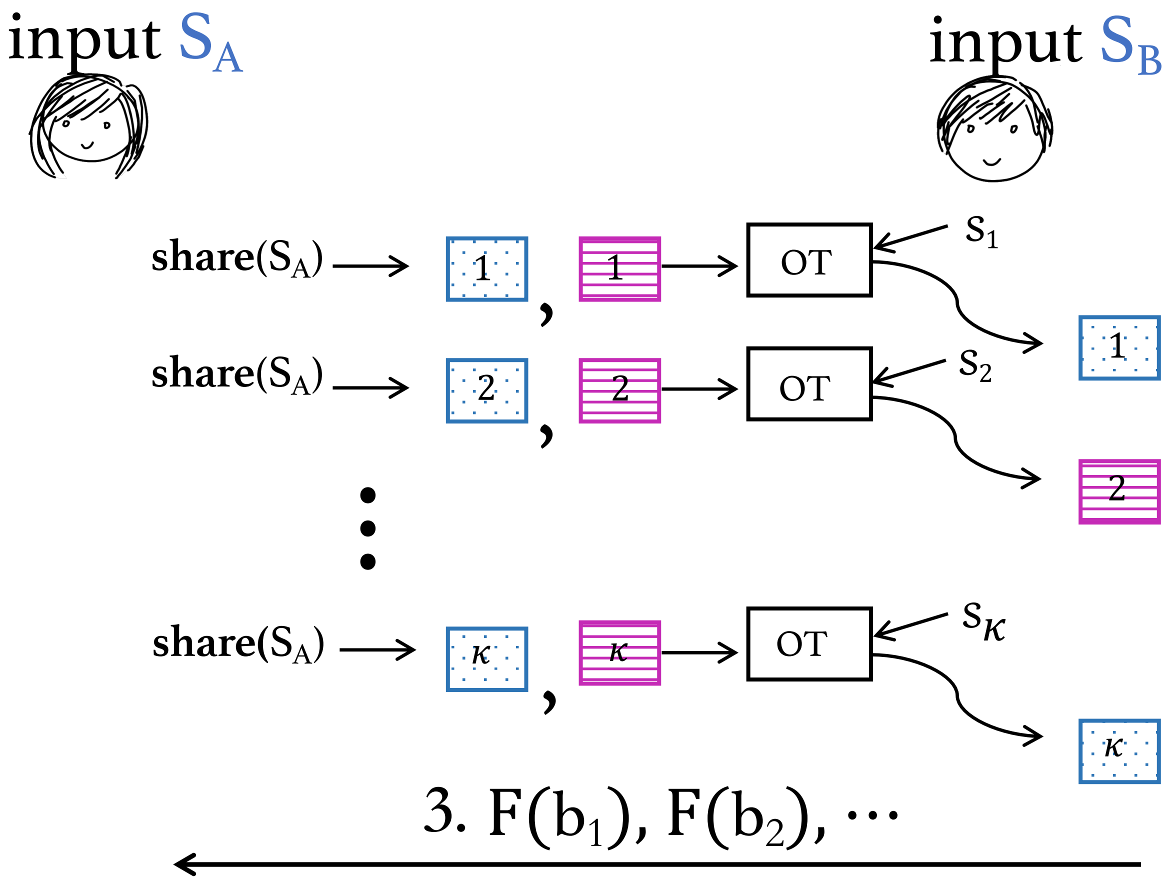


3. $F(b_1), F(b_2), \cdots$

$$F(x) = \mathbf{H}(\mathbf{ev}(\boxed{1}, x) \| \mathbf{ev}(\boxed{2}, x) \| \cdots \mathbf{ev}(\boxed{\kappa}, x))$$

How does the sa-PSI protocol work?



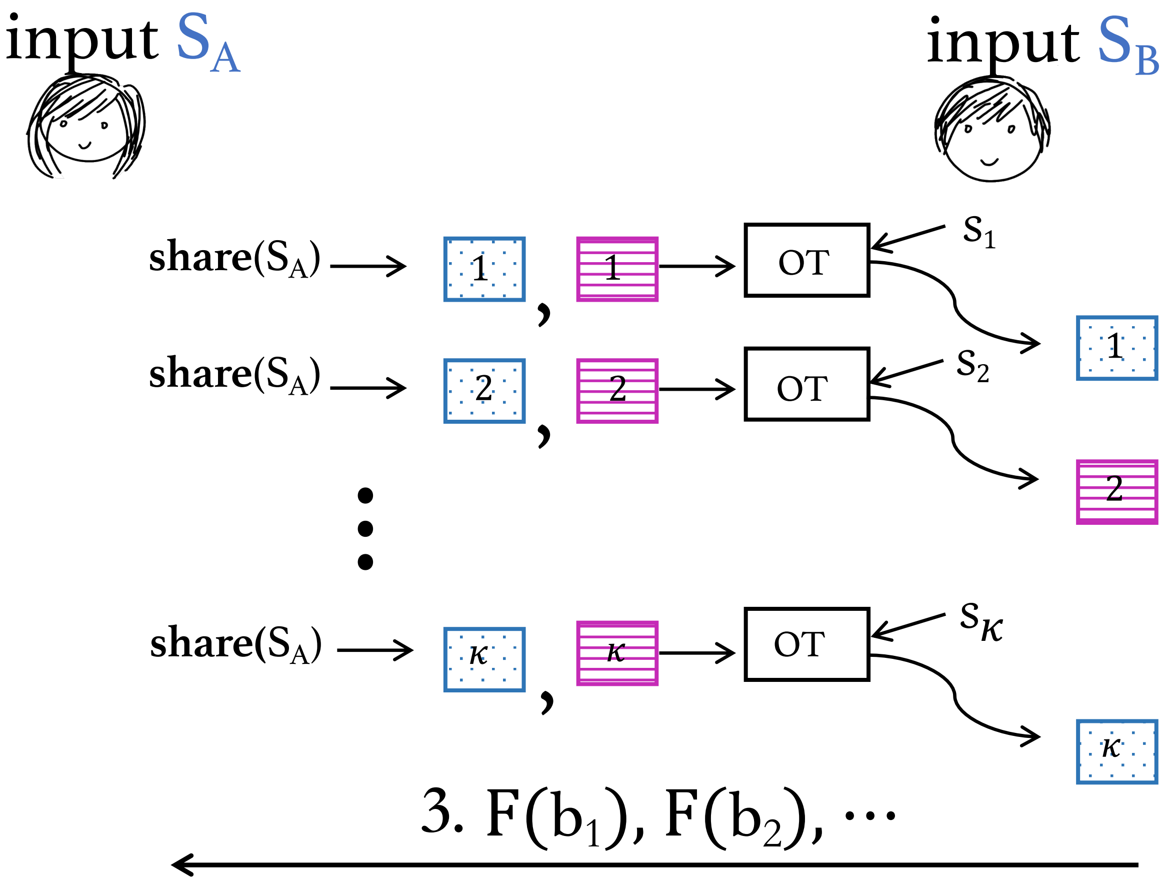
if $x \in S_A \Longrightarrow$ **ev**(▤ , x) = **ev**(⬚ , x)

if $x \in S_A \quad \Longrightarrow$ Alice can compute $F(x)$

if $x \notin S_A \quad \Longrightarrow F(x) \approx$ \$\$ looks random

$$F(x) = \mathbf{H}(\mathbf{ev}(\boxed{1}, x) \| \mathbf{ev}(\boxed{2}, x) \| \cdots \mathbf{ev}(\boxed{\kappa}, x))$$

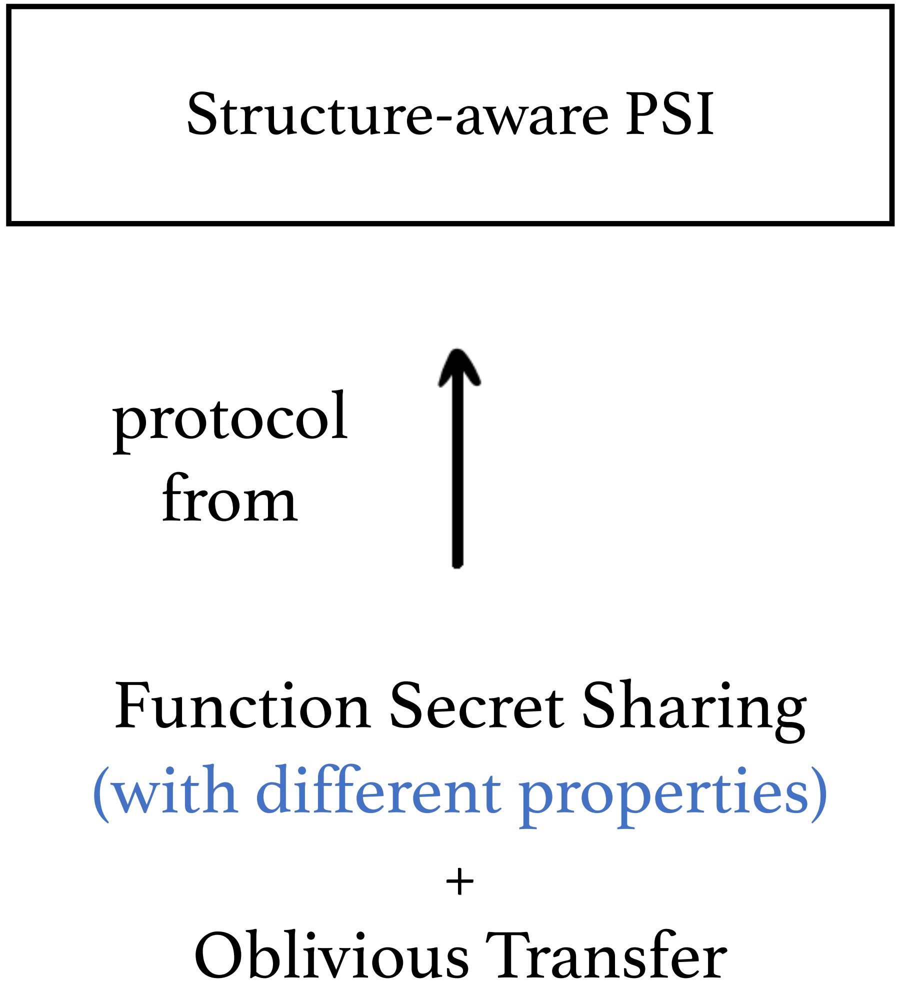
Full protocol



3. $\forall a \in S_A$, compute $F(a)$
4. locally compare to learn intersection

$$F(x) = \mathbf{H}(\mathbf{ev}(\boxed{1}, x) \| \mathbf{ev}(\boxed{2}, x) \| \cdots \mathbf{ev}(\boxed{\kappa}, x))$$

Summary



Structure-aware PSI

protocol
from

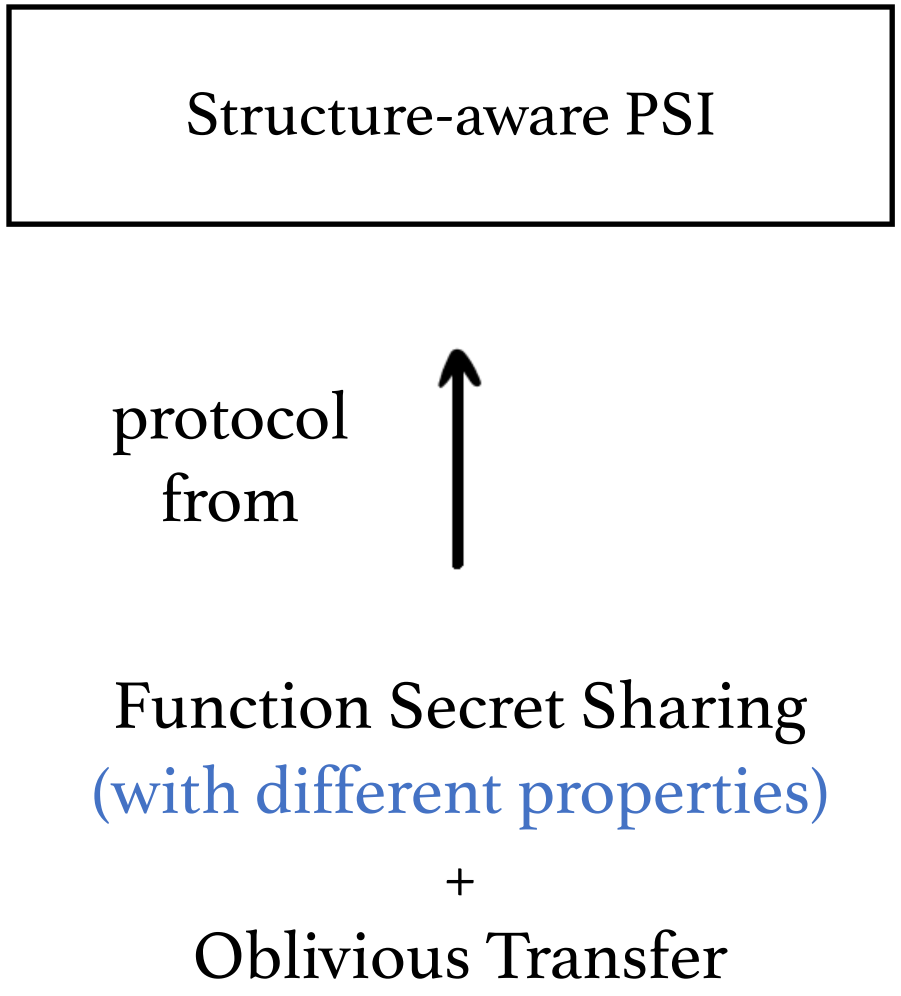
Function Secret Sharing
(with different properties)
+
Oblivious Transfer

- Formalized Structure Aware PSI
- Construct a general PSI framework from (variants of) Function Secret Sharing
  - ✓ semi-honest
  - ✓ malicious adversaries
  - ✓ comm + comp scale with description of structured set
- Formalize the properties of required FSS
- Present FSS constructions for {union of L_∞ balls metric space} set family

Future Directions

- Can we extend our techniques to other distance metrics like L2 norm, Hamming distance metrics?
- Can we construct FSS for other structures (motivated by other applications beyond fuzzy matching)?
- Can we improve the malicious framework to get comp and comm ~ description of set family?

Takeaway



Structure-aware PSI

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(with different properties)

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Oblivious Transfer