Private Set Intersection for Small Sets

Mike Rosulek, Oregon State University

NIST WPEC, September 24, 2024

Joint work with: Yeongjin Jang, Stanislav Lyakhov, Lawrence Roy, Ni Trieu

what is private set intersection (PSI)?

Alice

p x o

n r e

s u m

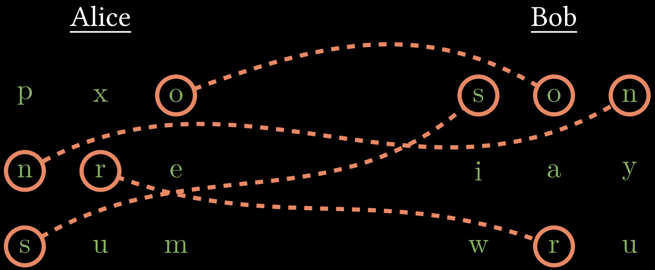
Bob

s o n

i a y

w r u

what is private set intersection (PSI)?



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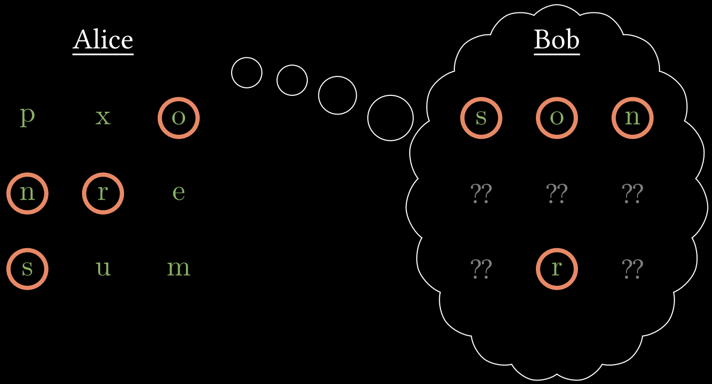

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p x o
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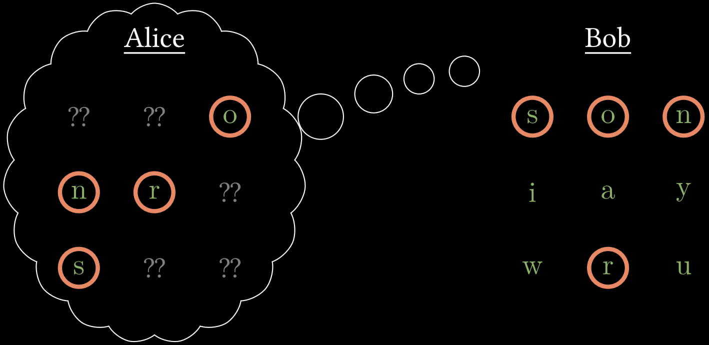
s u m

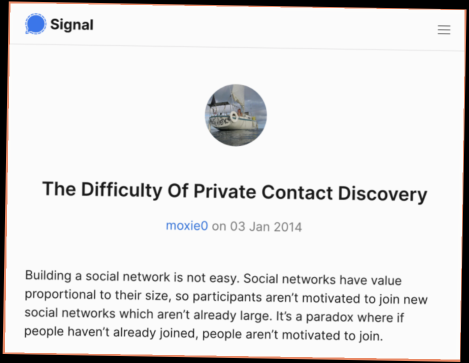
Bob

s o n
?? ?? ??
?? r ??

what is private set intersection (PSI)?



why use PSI?



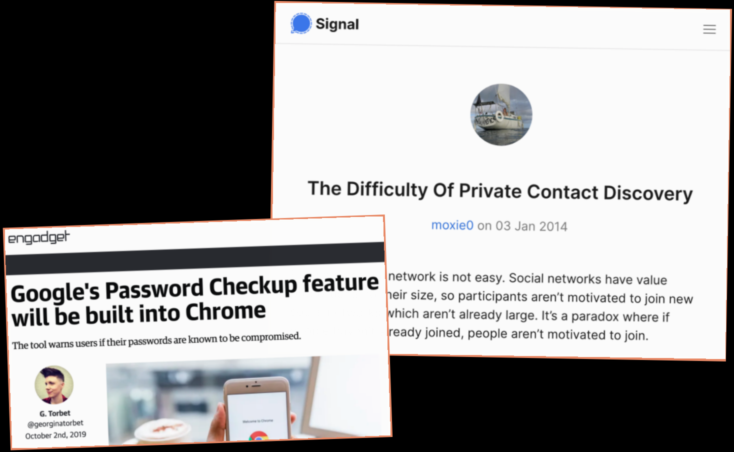
The Difficulty Of Private Contact Discovery

moxie0 on 03 Jan 2014

Building a social network is not easy. Social networks have value proportional to their size, so participants aren't motivated to join new social networks which aren't already large. It's a paradox where if people haven't already joined, people aren't motivated to join.

{my phone contacts} ∩ {users of your service}

why use PSI?



Signal

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engadget

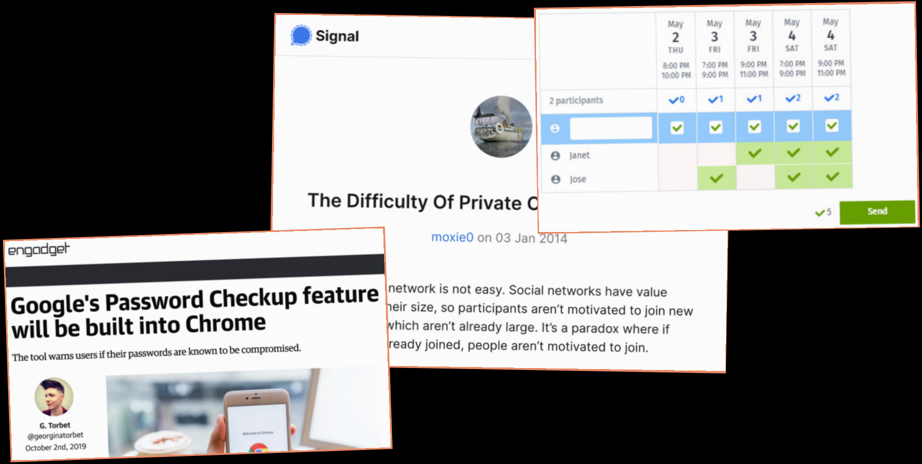
Google's Password Checkup feature will be built into Chrome

The tool warns users if their passwords are known to be compromised.

G. Torbet
@georginatorbet
October 2nd, 2019

{my passwords} ∩ {passwords found in breaches}

why use PSI?



Signal

The Difficulty Of Private C

moxie0 on 03 Jan 2014

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	May 2 THU 8:00 PM 10:00 PM	May 3 FRI 7:00 PM 9:00 PM	May 3 FRI 9:00 PM 11:00 PM	May 4 SAT 7:00 PM 9:00 PM	May 4 SAT 9:00 PM 11:00 PM
2 participants	✓0	✓1	✓1	✓2	✓2
	☑	☑	☑	☑	☑
Janet			✓	✓	✓
Jose		✓		✓	✓

✓5 Send

engadget

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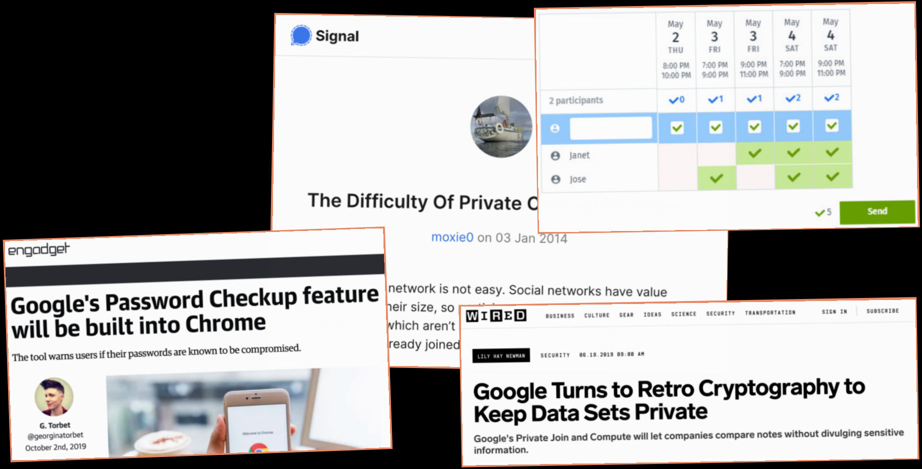
@georginatorbet
October 2nd, 2019

{my availability} ∩ {your availability}

why use PSI?



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October 2nd, 2019

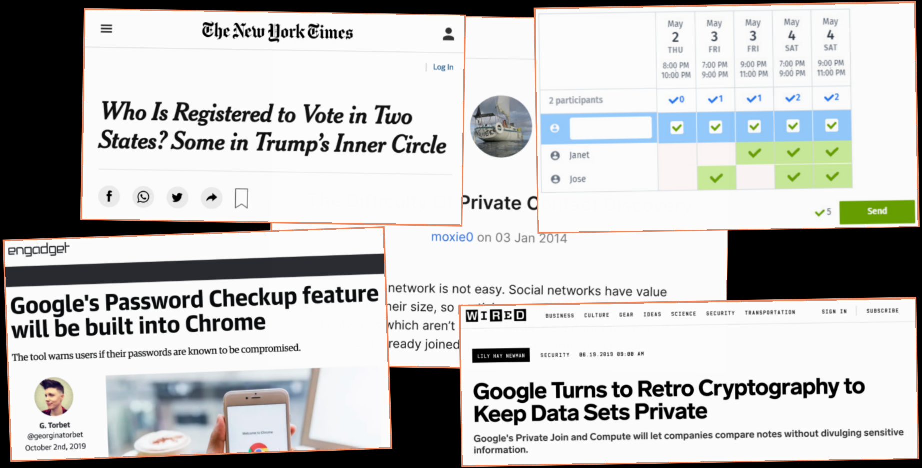
LILY HAY NEWMAN SECURITY 06.19.2019 09:00 AM

Google Turns to Retro Cryptography to Keep Data Sets Private

Google's Private Join and Compute will let companies compare notes without divulging sensitive information.

{people who saw ad} ∩ {customers who made purchases}

why use PSI?



The New York Times

Log In

Who Is Registered to Vote in Two States? Some in Trump's Inner Circle

Private C

moxie0 on 03 Jan 2014

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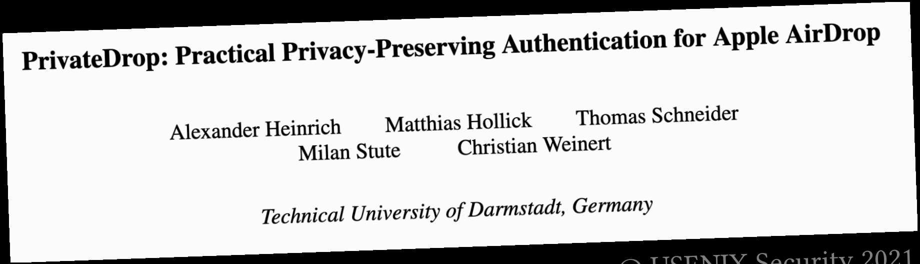
{voters registered in OR} ∩ {voters registered in NY}

motivating PSI for small sets

PSI for small sets = PSI for *personal privacy*

motivating PSI for small sets



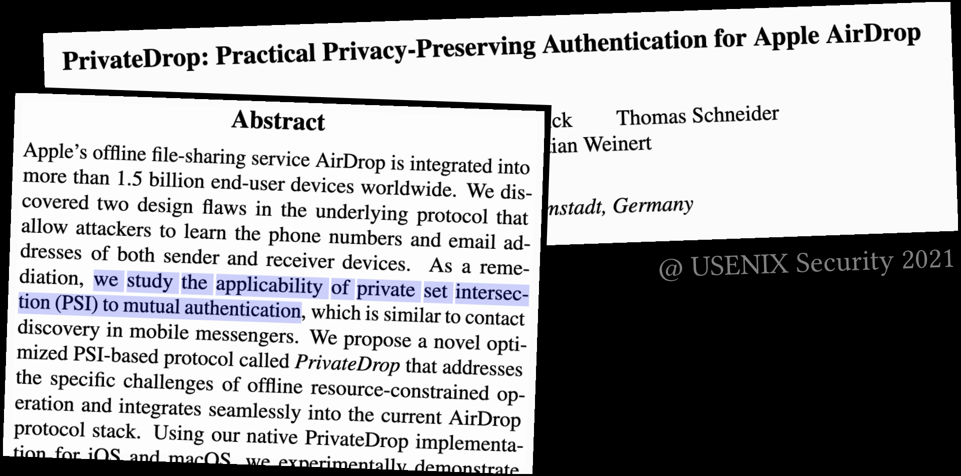
PrivateDrop: Practical Privacy-Preserving Authentication for Apple AirDrop

Alexander Heinrich Matthias Hollick Thomas Schneider
Milan Stute Christian Weinert

Technical University of Darmstadt, Germany

@ USENIX Security 2021

motivating PSI for small sets



PrivateDrop: Practical Privacy-Preserving Authentication for Apple AirDrop

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ian Weinert

nstadt, Germany

Abstract

Apple's offline file-sharing service AirDrop is integrated into more than 1.5 billion end-user devices worldwide. We discovered two design flaws in the underlying protocol that allow attackers to learn the phone numbers and email addresses of both sender and receiver devices. As a remediation, we study the applicability of private set intersection (PSI) to mutual authentication, which is similar to contact discovery in mobile messengers. We propose a novel optimized PSI-based protocol called *PrivateDrop* that addresses the specific challenges of offline resource-constrained operation and integrates seamlessly into the current AirDrop protocol stack. Using our native PrivateDrop implementation for iOS and macOS, we experimentally demonstrate

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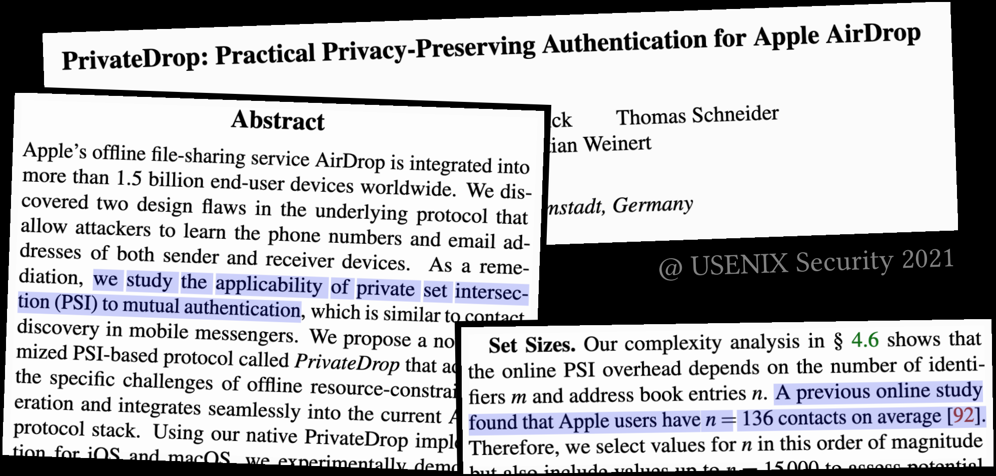
motivating PSI *for small sets*



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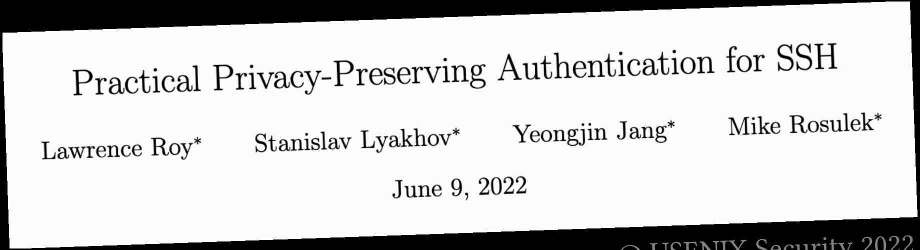
Set Sizes. Our complexity analysis in § 4.6 shows that the online PSI overhead depends on the number of identifiers *m* and address book entries *n*. A previous online study found that Apple users have $n = 136$ contacts on average [92]. Therefore, we select values for *n* in this order of magnitude but also include values up to $n = 15\,000$ to assess potential

[92] = Stute, Narain, Mariotto, Heinrich, Kreitschmann, Noubir, Hollick @ *USENIX* 2019

motivating PSI for small sets

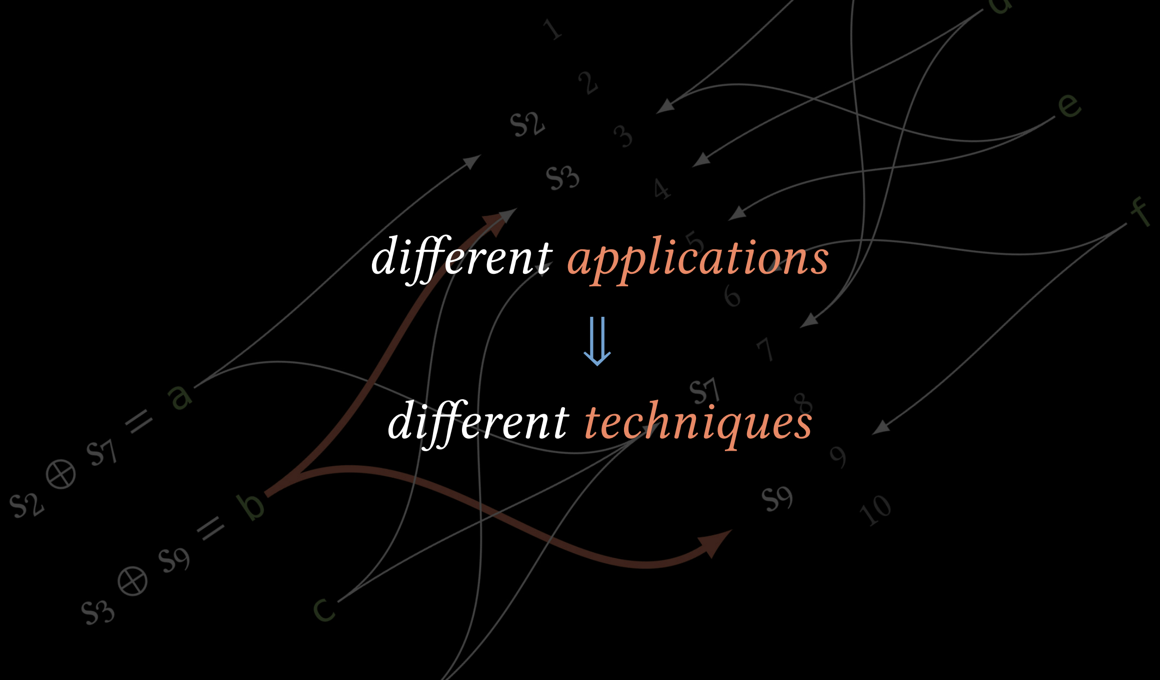


Practical Privacy-Preserving Authentication for SSH

Lawrence Roy* Stanislav Lyakhov* Yeongjin Jang* Mike Rosulek*

June 9, 2022

@ USENIX Security 2022



different applications

⇓

different techniques

PSI on **small sets** (hundreds)

- private availability poll
- key agreement techniques

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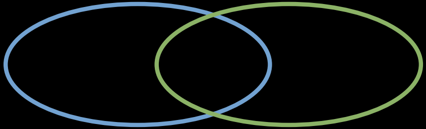
- key agreement techniques

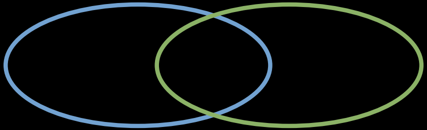


PSI on **large sets** (millions)

- double-registered voters
- OT extension; combinatorial tricks

PSI on **small sets** (hundreds)

- private availability poll
- key agreement techniques



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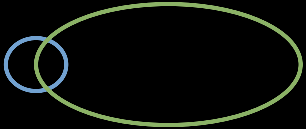
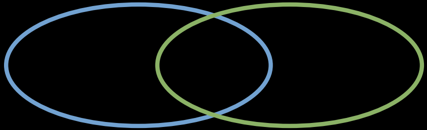


PSI on **asymmetric sets** (100 : billion)

- contact discovery; password checkup
- offline phase; leakage

PSI on **small sets** (hundreds)

- private availability poll
- key agreement techniques



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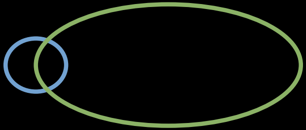


PSI on **asymmetric sets** (100 : billion)

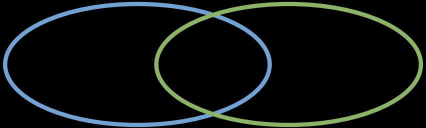
- contact discovery; password checkup
- offline phase; leakage

computing on the intersection

- sales statistics about intersection
- generic MPC

PSI on **small sets** (hundreds)

- private availability poll
- key agreement techniques

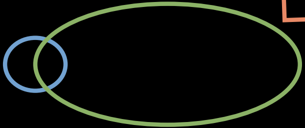


PSI on **large sets** (millions)

- oters
- inatorial tricks

Not to mention:

- approximate/fuzzy matching
- more than 2 parties/sets
- private set *union*



PSI on **asymmetric sets** (100 : billion)

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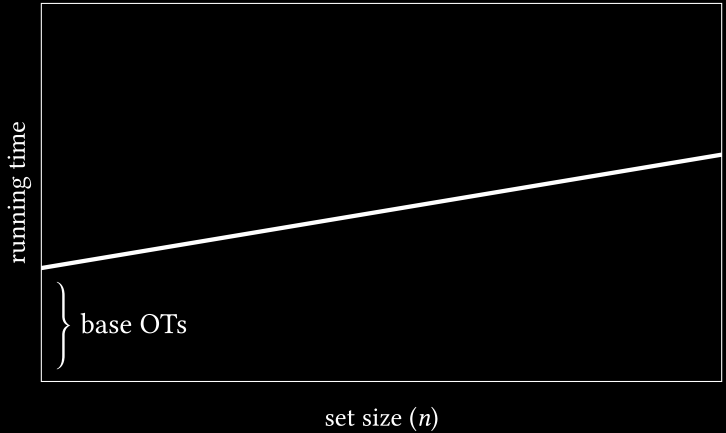
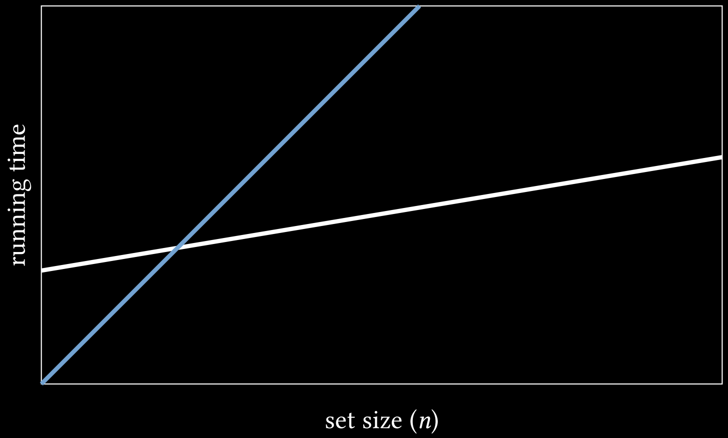
PSI techniques for small sets



OT-based PSI:

- 128 base OTs
- $O(n)$ symm-key ops

PSI techniques for small sets



OT-based PSI:

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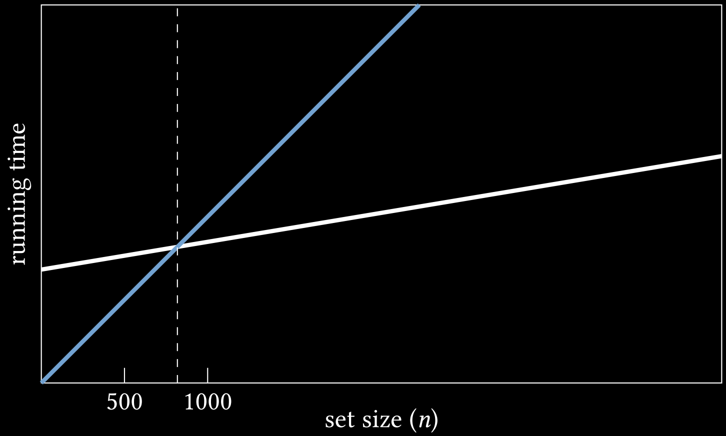
KA-based PSI:

- $O(n)$ pub-key ops

PSI techniques for small sets



OT-based PSI:

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KA-based PSI:

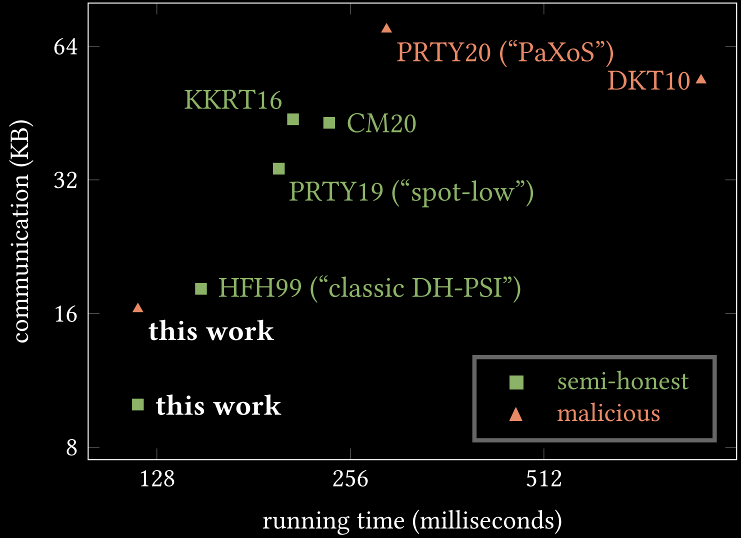
- $O(n)$ pub-key ops

Compact and Malicious *Private Set Intersection* for Small Sets

Mike Rosulek, Oregon State University
Ni Trieu, Arizona State University

appeared at ACM CCS 2021

PSI cost: 256 items per party:



our **semi-honest** PSI:

- 45% ↓ communication
- 20% ↓ runtime

our **malicious** PSI:

- 10% ↓ communication
- 20% ↓ runtime

vs. best *semi-honest* PSI!

- 75% ↓ communication
- 55% ↓ runtime

vs. best malicious PSI

Alice (random oracle $H : \{0,1\}^* \to \mathbb{G}$) **Bob**

$x_1, x_2, \ldots$

$y_1, y_2, \ldots$

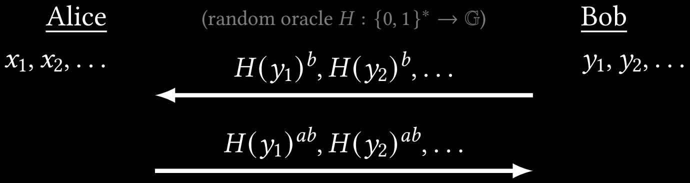
[HubermanFranklinHogg99]

Alice (random oracle $H : \{0,1\}^* \longrightarrow \mathbb{G}$) **Bob**

$x_1, x_2, \ldots$ $y_1, y_2, \ldots$

$$\xleftarrow{\quad H(y_1)^b, H(y_2)^b, \ldots \quad}$$

[HubermanFranklinHogg99]



Alice

(random oracle $H : \{0,1\}^* \rightarrow \mathbb{G}$)

Bob

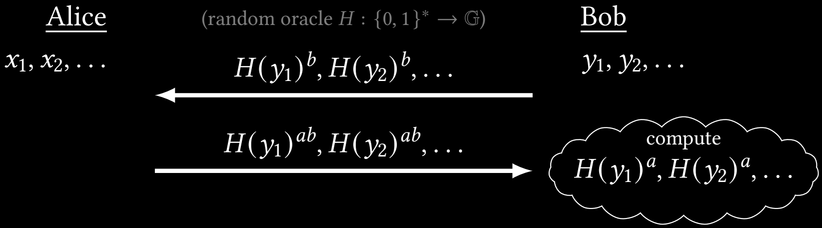
$x_1, x_2, \ldots$

$y_1, y_2, \ldots$

$H(y_1)^b, H(y_2)^b, \ldots$ ←

$H(y_1)^{ab}, H(y_2)^{ab}, \ldots$ →

[HubermanFranklinHogg99]



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(random oracle $H : \{0,1\}^* \to \mathbb{G}$)

Bob

$x_1, x_2, \ldots$

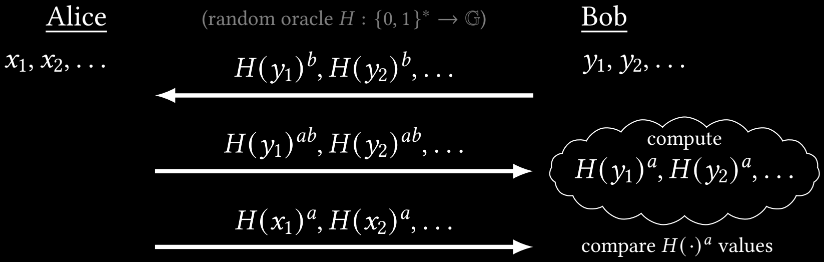
$H(y_1)^b, H(y_2)^b, \ldots$

$y_1, y_2, \ldots$

$H(y_1)^{ab}, H(y_2)^{ab}, \ldots$

compute
$H(y_1)^a, H(y_2)^a, \ldots$

[HubermanFranklinHogg99]



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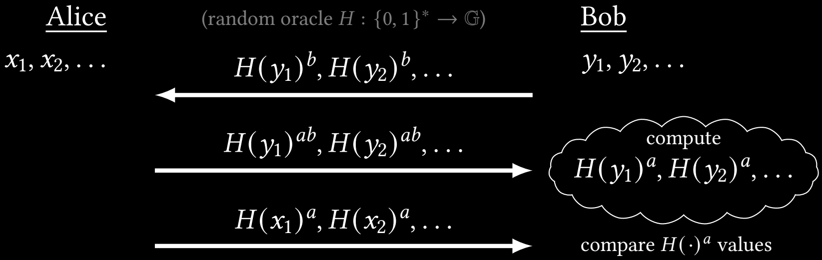
$H(y_1)^{ab}, H(y_2)^{ab}, \ldots$

compute $H(y_1)^a, H(y_2)^a, \ldots$

$H(x_1)^a, H(x_2)^a, \ldots$

compare $H(\cdot)^a$ values

[HubermanFranklinHogg99]



Alice (random oracle $H : \{0,1\}^* \to \mathbb{G}$) **Bob**

$x_1, x_2, \ldots$

$y_1, y_2, \ldots$

$H(y_1)^b, H(y_2)^b, \ldots$

$H(y_1)^{ab}, H(y_2)^{ab}, \ldots$

compute $H(y_1)^a, H(y_2)^a, \ldots$

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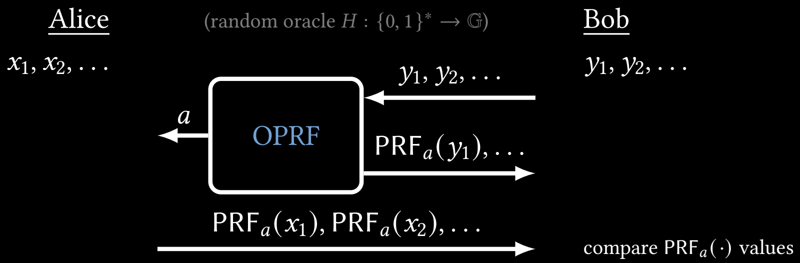
compare $H(\cdot)^a$ values

Semi-honest security:

- $x \mapsto H(x)^a$ is a PRF (DDH assumption + random oracle)
- first two messages are an oblivious PRF protocol

[HubermanFranklinHogg99]



Alice (random oracle $H : \{0,1\}^* \to \mathbb{G}$) Bob

$x_1, x_2, \ldots$

$y_1, y_2, \ldots$

OPRF

a

$y_1, y_2, \ldots$

$\mathsf{PRF}_a(y_1), \ldots$

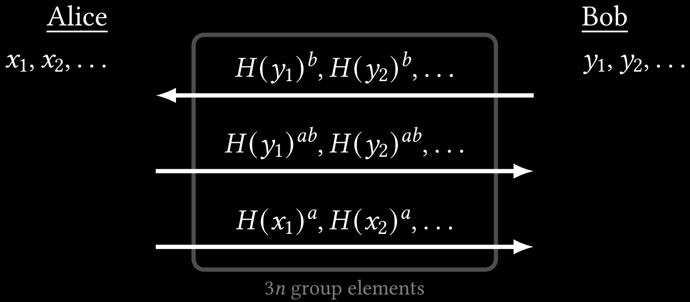
$\mathsf{PRF}_a(x_1), \mathsf{PRF}_a(x_2), \ldots$

compare $\mathsf{PRF}_a(\cdot)$ values

Semi-honest security:

- $x \mapsto H(x)^a$ is a PRF (DDH assumption + random oracle)
- first two messages are an oblivious PRF protocol
- standard OPRF→PSI paradigm [FreedmanIshaiPinkasReingold05]

[HubermanFranklinHogg99]



Alice

$x_1, x_2, \ldots$

Bob

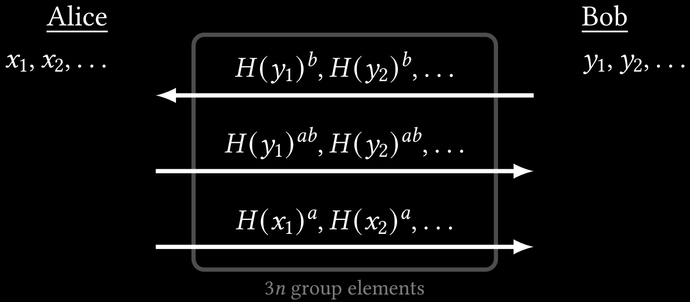
$y_1, y_2, \ldots$

$H(y_1)^b, H(y_2)^b, \ldots$

$H(y_1)^{ab}, H(y_2)^{ab}, \ldots$

$H(x_1)^a, H(x_2)^a, \ldots$

$3n$ group elements



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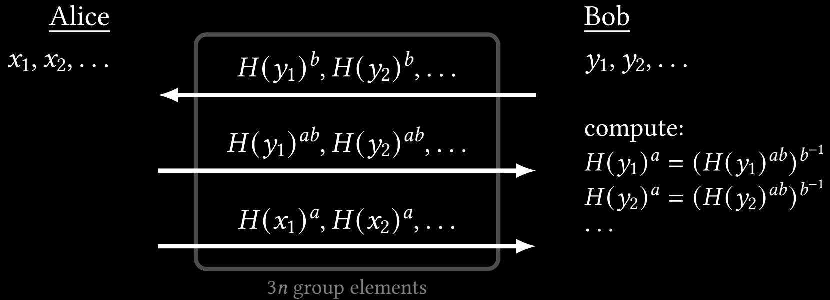
$H(x_1)^a, H(x_2)^a, \ldots$

$3n$ group elements

how could you possibly reduce communication?



Alice

$x_1, x_2, \ldots$

Bob

$y_1, y_2, \ldots$

$H(y_1)^b, H(y_2)^b, \ldots$

$H(y_1)^{ab}, H(y_2)^{ab}, \ldots$

$H(x_1)^a, H(x_2)^a, \ldots$

compute:

$H(y_1)^a = (H(y_1)^{ab})^{b^{-1}}$

$H(y_2)^a = (H(y_2)^{ab})^{b^{-1}}$

$\ldots$

$3n$ group elements

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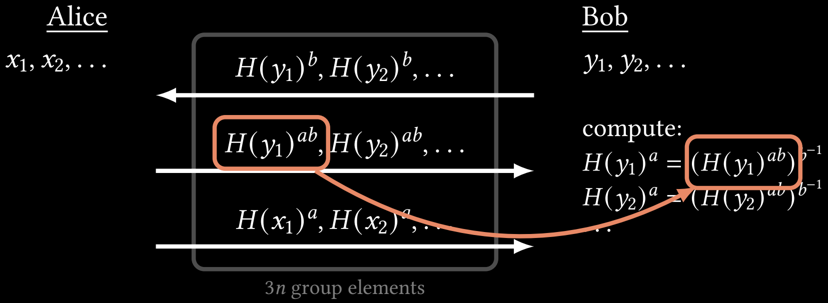


Alice

$x_1, x_2, \ldots$

Bob

$y_1, y_2, \ldots$

$H(y_1)^b, H(y_2)^b, \ldots$

$\boxed{H(y_1)^{ab}}, H(y_2)^{ab}, \ldots$

$H(x_1)^a, H(x_2)^a, \ldots$

3n group elements

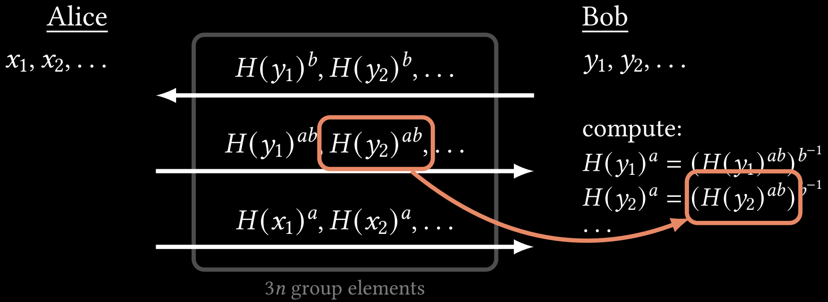
compute:

$H(y_1)^a = \boxed{(H(y_1)^{ab})}^{b^{-1}}$

$H(y_2)^a = (H(y_2)^{ab})^{b^{-1}}$

$\ldots$

how could you possibly reduce communication?



Alice

$x_1, x_2, \ldots$

$H(y_1)^b, H(y_2)^b, \ldots$

$H(y_1)^{ab}, \boxed{H(y_2)^{ab}}, \ldots$

$H(x_1)^a, H(x_2)^a, \ldots$

$3n$ group elements

Bob

$y_1, y_2, \ldots$

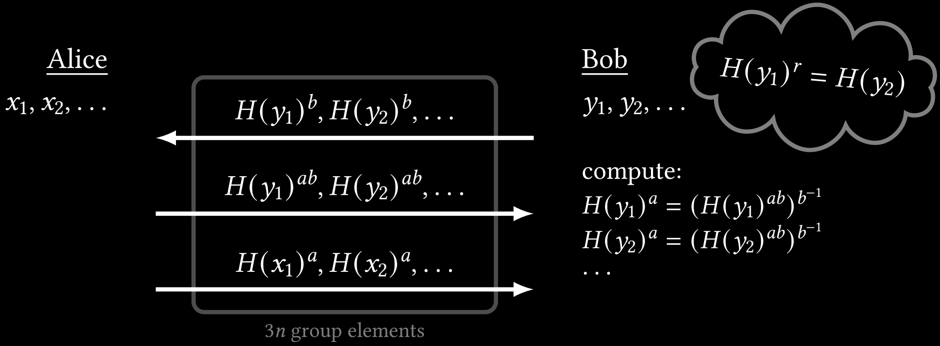
compute:

$H(y_1)^a = (H(y_1)^{ab})^{b^{-1}}$

$H(y_2)^a = \boxed{(H(y_2)^{ab})}^{b^{-1}}$

$\ldots$

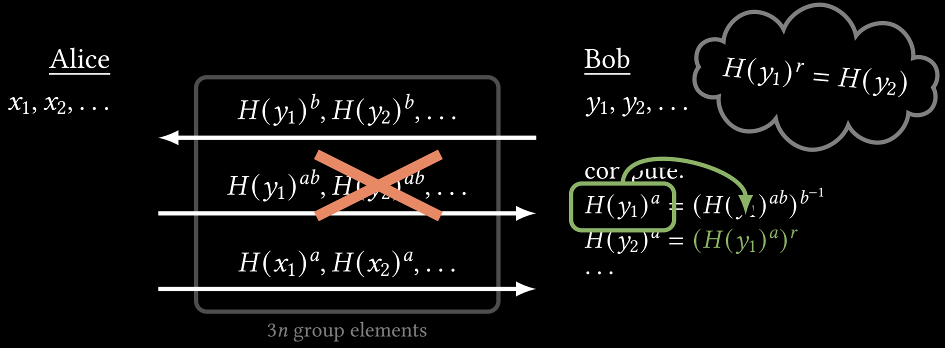
how could you possibly reduce communication?



how could you possibly reduce communication?

replace random oracle with some "trapdoored" function

… where Bob knows dlog relationships between outputs



Alice

$x_1, x_2, \ldots$

Bob

$y_1, y_2, \ldots$

$H(y_1)^r = H(y_2)$

$H(y_1)^b, H(y_2)^b, \ldots$

$\xcancel{H(y_1)^{ab}, H(y_2)^{ab}, \ldots}$

$H(x_1)^a, H(x_2)^a, \ldots$

3*n* group elements

compute:

$\boxed{H(y_1)^a} = (H(y_1)^{ab})^{b^{-1}}$

$H(y_2)^a = (H(y_1)^a)^r$

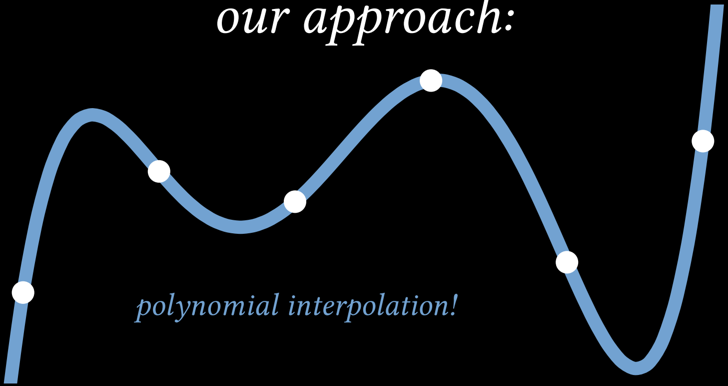
$\ldots$

how could you possibly reduce communication?

replace random oracle with some "trapdoored" function

… where Bob knows dlog relationships between outputs

our approach:



polynomial interpolation!

Alice	Bob
$x_1, x_2, \ldots$	$y_1, y_2, \ldots$

Alice

$x_1, x_2, \ldots$

Bob

$y_1, y_2, \ldots$

interpolate poly P:

$P(y_i) = g^{b_i}$

Alice

$x_1, x_2, \ldots$

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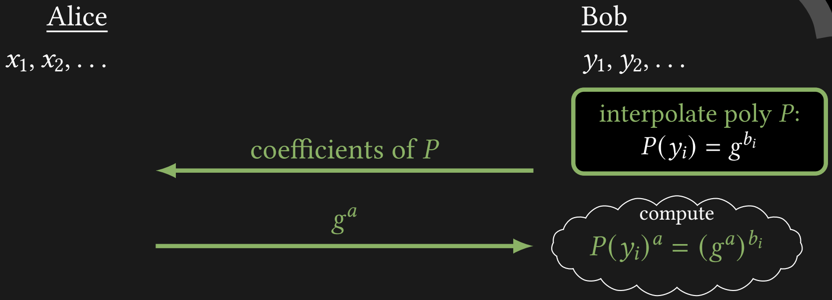
coefficients of P



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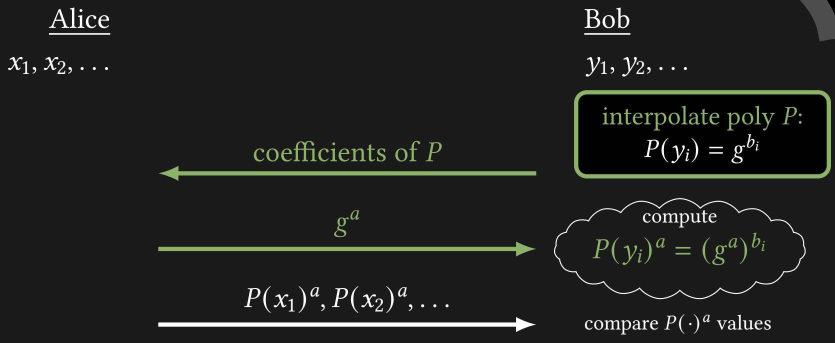

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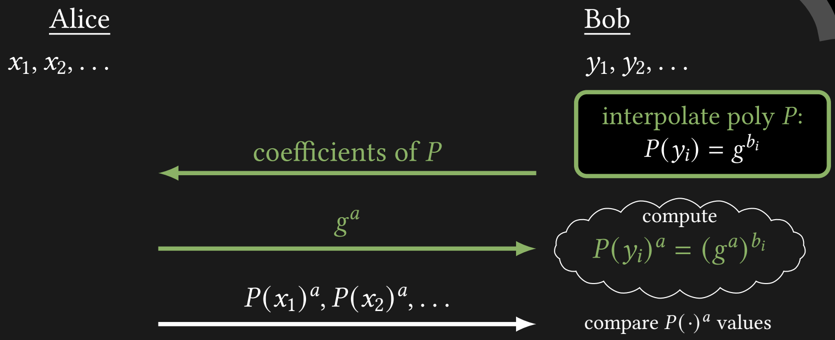
$P(x_1)^a, P(x_2)^a, \ldots$

compare $P(\cdot)^a$ values



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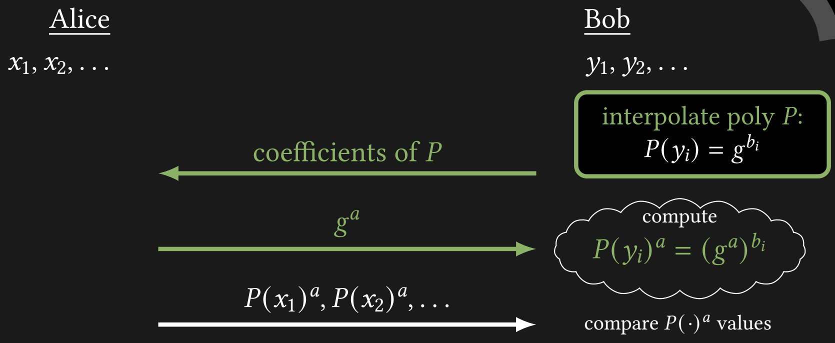
correctness: Bob knows dlog of $P(y)$ for programmed points ✓



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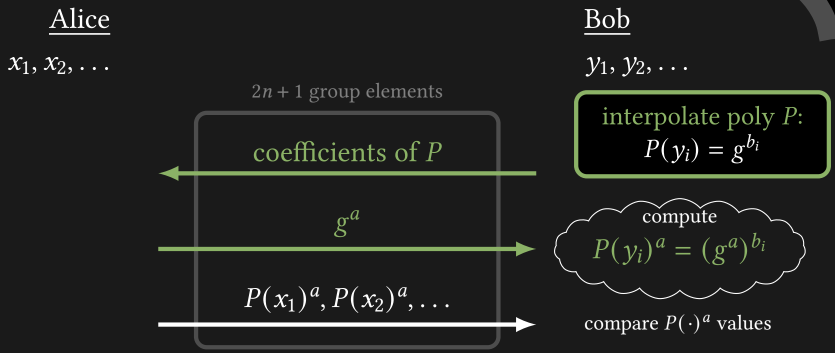

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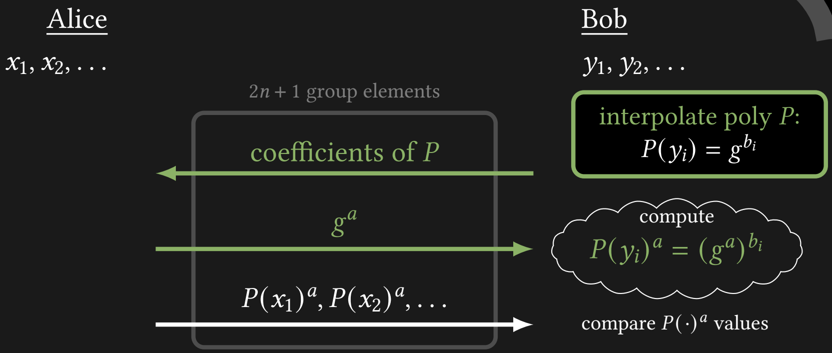
obliviousness:	description of P doesn't leak choice of programmed points ✓
efficiency:	$\lvert$description of $P\rvert = n$ group elements ✓



Alice

$x_1, x_2, \ldots$

Bob

$y_1, y_2, \ldots$

interpolate poly P:
$P(y_i) = g^{b_i}$

$2n + 1$ group elements

coefficients of P ←

g^a →

$P(x_1)^a, P(x_2)^a, \ldots$ →

compute
$P(y_i)^a = (g^a)^{b_i}$

compare $P(\cdot)^a$ values

correctness: Bob knows dlog of $P(y)$ for programmed points ✓

obliviousness: description of P doesn't leak choice of programmed points ✓

efficiency: $|\text{description of } P| = n$ group elements ✓

$P(\cdot)^a$ is PRF: Bob **cannot know** dlog of any *other* $P(x)$??

interpolate so that:

$$P(y_i) = g^{b_i}$$

$$??\Downarrow??$$

other $P(x)$ outputs
have unknown dlog

interpolate so that:

$P(y_i) = g^{b_i}$

$? ? \Downarrow ? ?$

other $P(x)$ outputs
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Ideal permutation model: all parties have oracle access to random Π, Π^{-1}

interpolate so that:
$P(y_i) = g^{b_i}$

? ? $\Downarrow$? ?

other $P(x)$ outputs
have unknown dlog

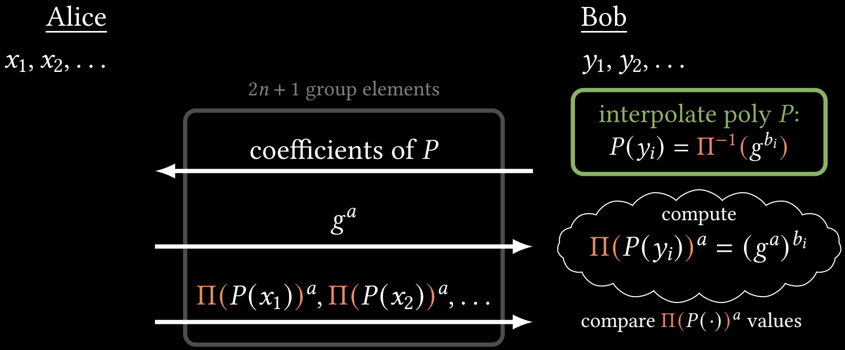
interpolate so that:
$P(y_i) = \Pi^{-1}(g^{b_i})$

$\Downarrow$ ✓

simulator can **program**
other $\Pi(P(x))$ outputs

Ideal permutation model: all parties have oracle access to random Π, Π^{-1}

our real protocol:



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$2n + 1$ group elements

coefficients of P

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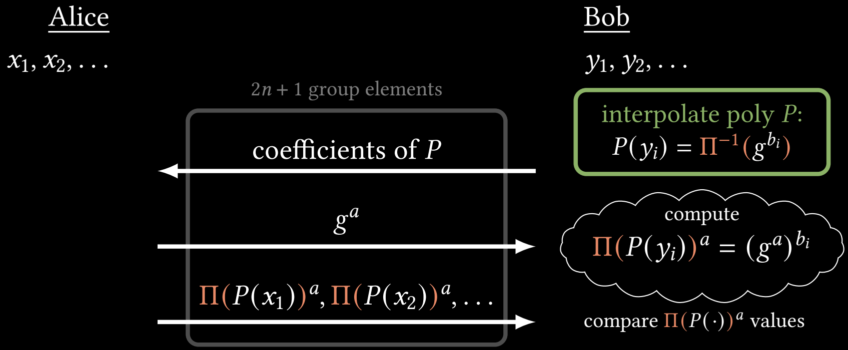
$\Pi(P(x_1))^a, \Pi(P(x_2))^a, \ldots$

compute

$\Pi(P(y_i))^a = (g^a)^{b_i}$

compare $\Pi(P(\cdot))^a$ values

our real protocol (fine print):



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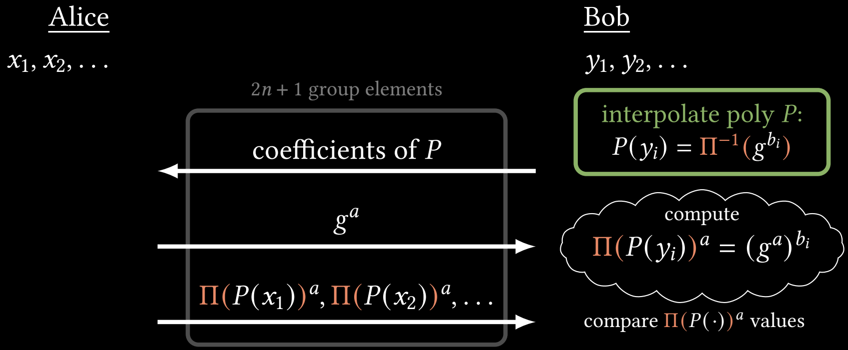

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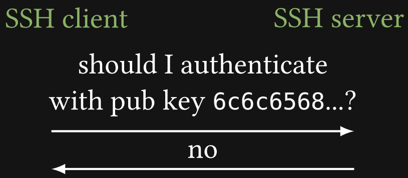
semi-honest: Alice's group elements can be truncated

malicious: a few more strategic RO calls (to help simulator extract)

Practical Privacy-Preserving Authentication for SSH

Lawrence Roy
Stanislav Lyakhov
Yeongjin Jang
Mike Rosulek

appeared at USENIX Security 2022

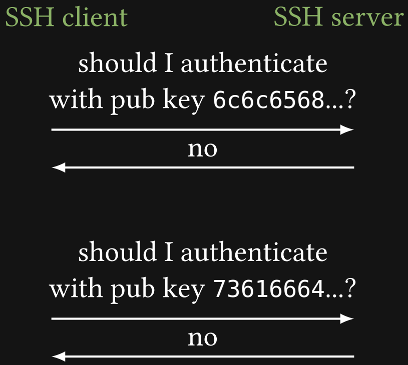


SSH client

SSH server

should I authenticate
with pub key `6c6c6568`…?

no



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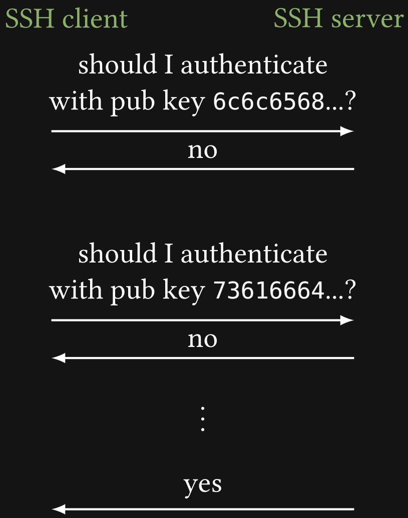

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⋮

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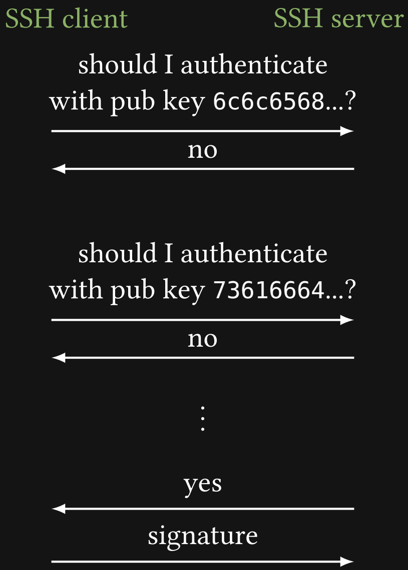

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⋮

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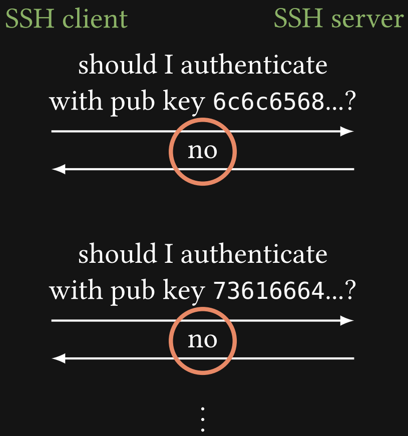
signature



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shou
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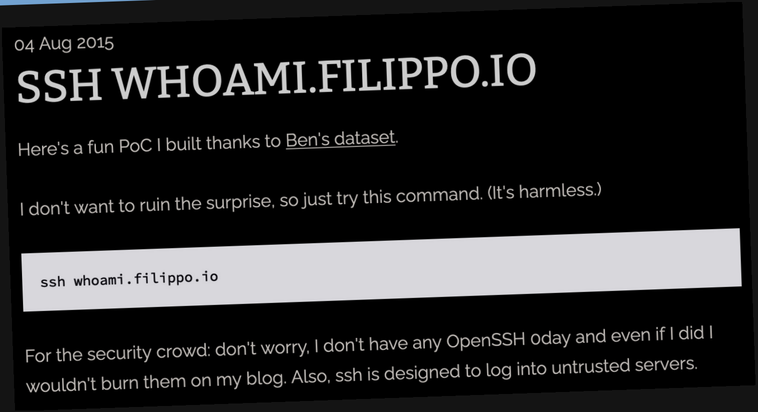


04 Aug 2015

SSH WHOAMI.FILIPPO.IO

Here's a fun PoC I built thanks to Ben's dataset.

I don't want to ruin the surprise, so just try this command. (It's harmless.)

```
ssh whoami.filippo.io
```

For the security crowd: don't worry, I don't have any OpenSSH 0day and even if I did I wouldn't burn them on my blog. Also, ssh is designed to log into untrusted servers.

Filippo Valsorda `https://words.filippo.io/ssh-whoami-filippo-io/`

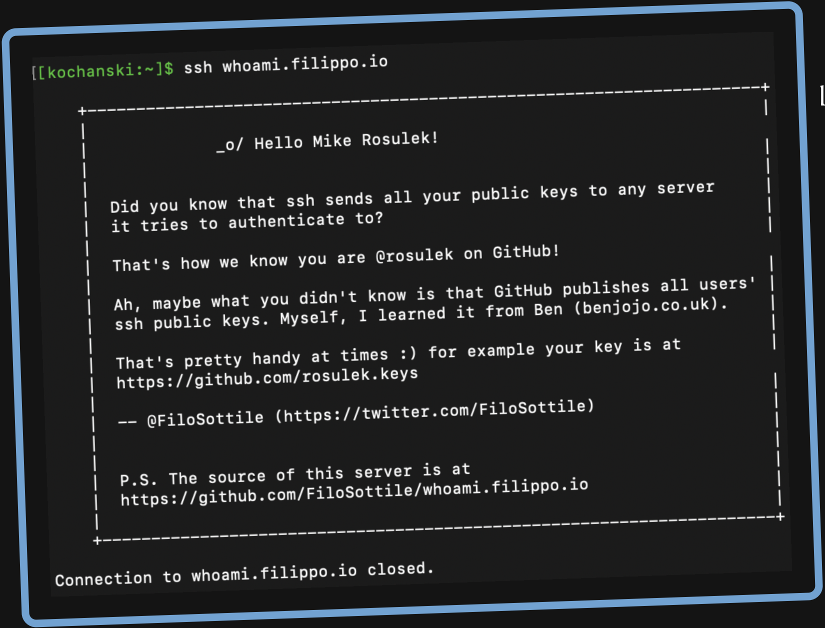
SSH client

shou

with p

shou

with pu



```
[[kochanski:~]$ ssh whoami.filippo.io

    +---------------------------------------------------------------------------+
    |                                                                           |
    |           _o/ Hello Mike Rosulek!                                         |
    |                                                                           |
    |                                                                           |
    |  Did you know that ssh sends all your public keys to any server           |
    |  it tries to authenticate to?                                             |
    |                                                                           |
    |  That's how we know you are @rosulek on GitHub!                           |
    |                                                                           |
    |  Ah, maybe what you didn't know is that GitHub publishes all users'       |
    |  ssh public keys. Myself, I learned it from Ben (benjojo.co.uk).          |
    |                                                                           |
    |  That's pretty handy at times :) for example your key is at               |
    |  https://github.com/rosulek.keys                                          |
    |                                                                           |
    |  -- @FiloSottile (https://twitter.com/FiloSottile)                        |
    |                                                                           |
    |                                                                           |
    |  P.S. The source of this server is at                                     |
    |  https://github.com/FiloSottile/whoami.filippo.io                         |
    |                                                                           |
    +---------------------------------------------------------------------------+

Connection to whoami.filippo.io closed.
```

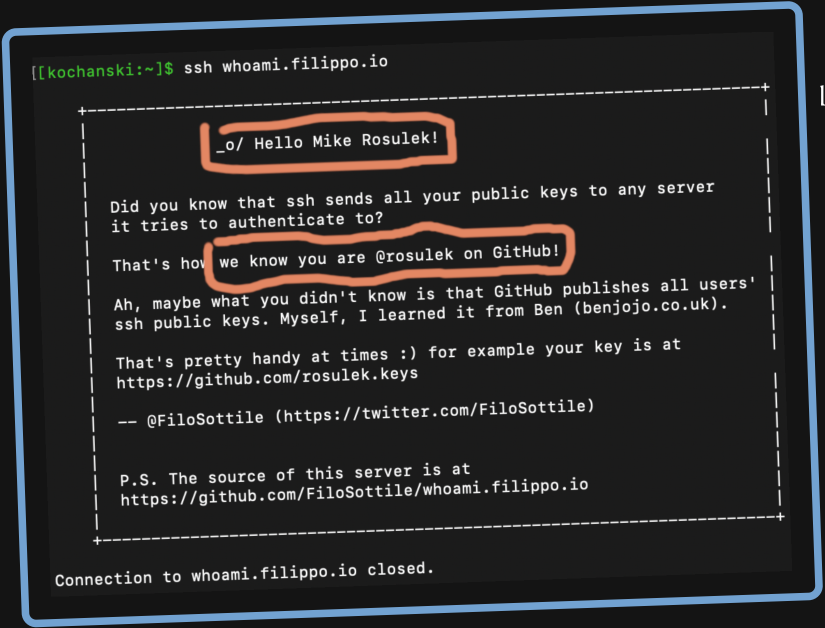
l keys

SSH client

shou
with p

shou
with pu

l keys



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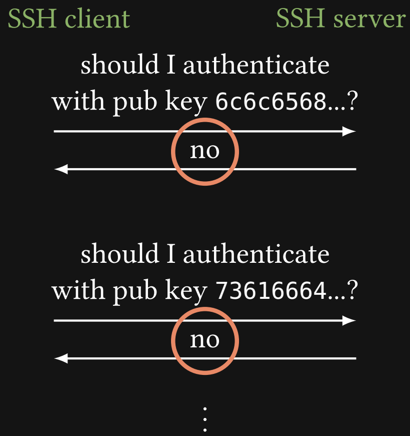

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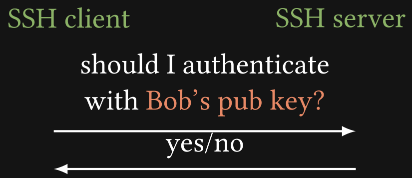
- refuse all advertisements ⇒ learn all keys
- can configure client to send only “correct” key



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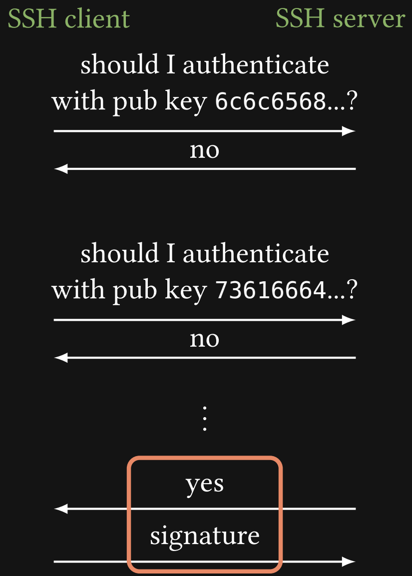
- offer someone else’s pub key, observe response
- *pre-emptive* signatures possible (in principle)



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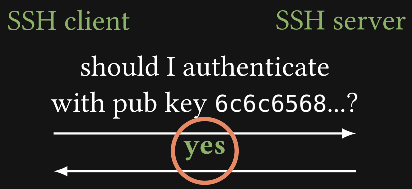
problem: server sees which key was used:

- and can **prove it!** ⇒ authentication not deniable
- fundamental to protocol



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problem: server sees which key was used:

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- fundamental to protocol

problem: server can act as honeypot:

- accept *any* key, even ones never seen before
- fundamental to protocol

goals of this work

1 server & client should learn minimal information

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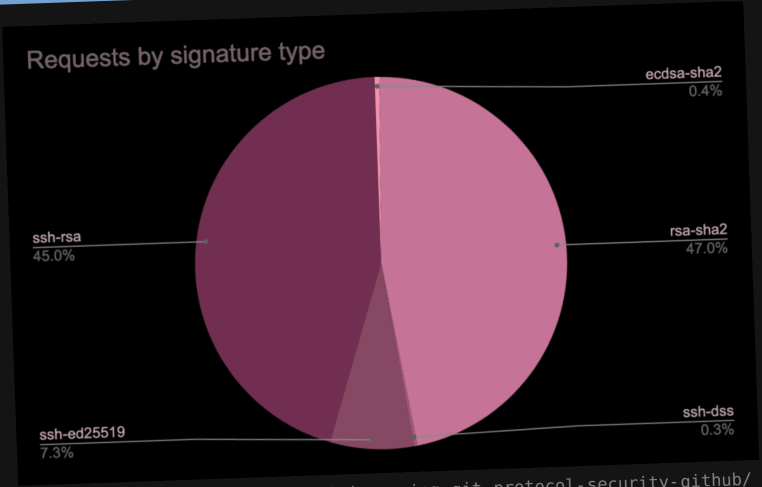
2 authenticate with respect to existing SSH keys

goals of this work

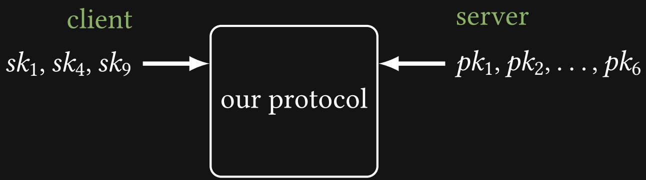


https://github.blog/2021-09-01-improving-git-protocol-security-github/

goals of this work

1. server & client should learn minimal information
2. authenticate with respect to existing SSH keys
3. minimize reliance on per-site configuration

our new authentication method: big picture



client

sk_1, sk_4, sk_9

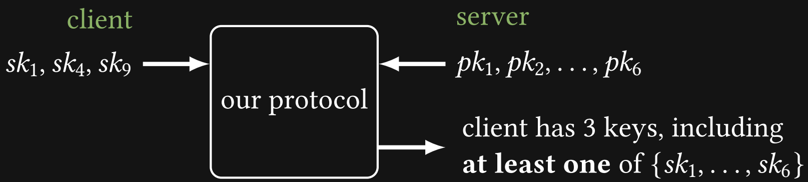
our protocol

server

$pk_1, pk_2, \ldots, pk_6$

- any **mixture** of existing RSA, ECDSA, EdDSA keys, in a single authentication attempt

our new authentication method: big picture



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client has 3 keys, including **at least one** of $\{sk_1, \ldots, sk_6\}$

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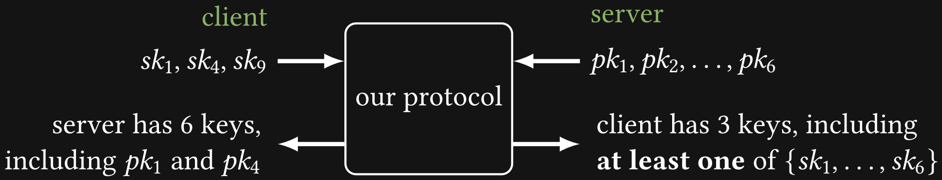

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sk_1, sk_4, sk_9

server

$pk_1, pk_2, \ldots, pk_6$

our protocol

server has 6 keys, including pk_1 and pk_4

client has 3 keys, including **at least one** of $\{sk_1, \ldots, sk_6\}$

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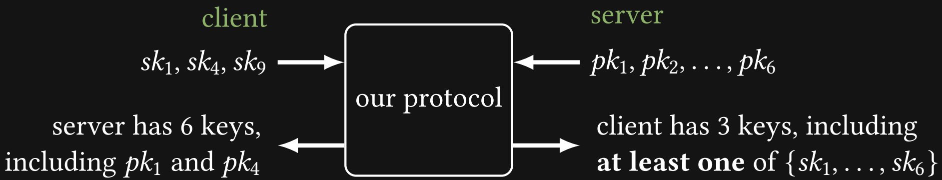

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- does not depend on site-specific configuration; safe to use **all keys** in every authentication attempts

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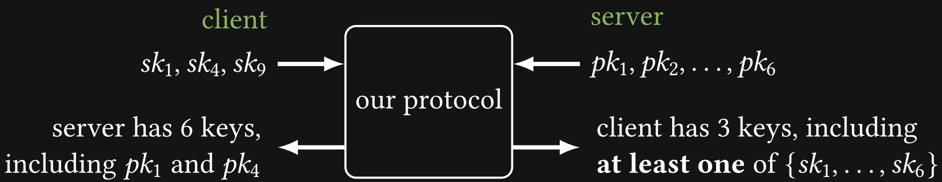

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our protocol

server has 6 keys,
including pk_1 and pk_4

client has 3 keys, including
at least one of $\{sk_1, \ldots, sk_6\}$

- any **mixture** of existing RSA, ECDSA, EdDSA keys, in a single authentication attempt
- does not depend on site-specific configuration; safe to use **all keys** in every authentication attempts
- client won't connect unless server **knows** and **explicitly includes** one of client's keys

technical overview

client (with $\{sk_i\}_i$):

server (with $\{pk_j\}_j$):

technical overview

client (with $\{sk_i\}_i$):

server (with $\{pk_j\}_j$):

$$c, \{m_j\}_j \leftarrow \mathsf{Enc}\left(\{pk_j\}_j\right)$$

1. anonymous multi-KEM

address ciphertext to $\{pk_j\}_j$;
sk_j decrypts c to m_j;
c hides pk_j recipients

technical overview

client (with $\{sk_i\}_i$):

server (with $\{pk_j\}_j$):

$\xleftarrow{c}$

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$$c, \{m_j\}_j \leftarrow \mathsf{Enc}\Big(\{pk_j\}_j\Big)$$

$$\xleftarrow{\;c\;}$$

$$\Big\{\widehat{m}_i := \mathsf{Dec}(sk_i, c)\Big\}_i$$

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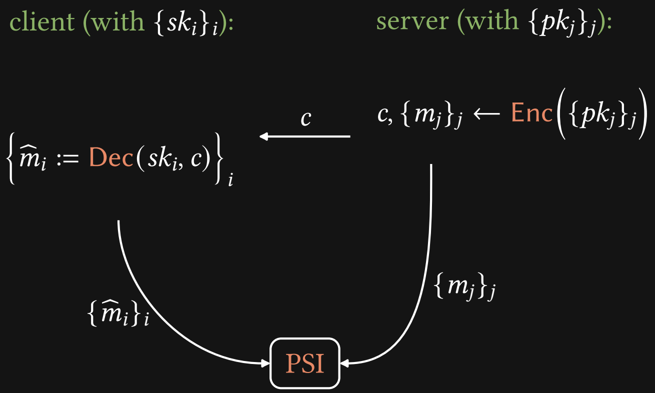
technical overview



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$\{m_j\}_j$

PSI

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each party has set of items;

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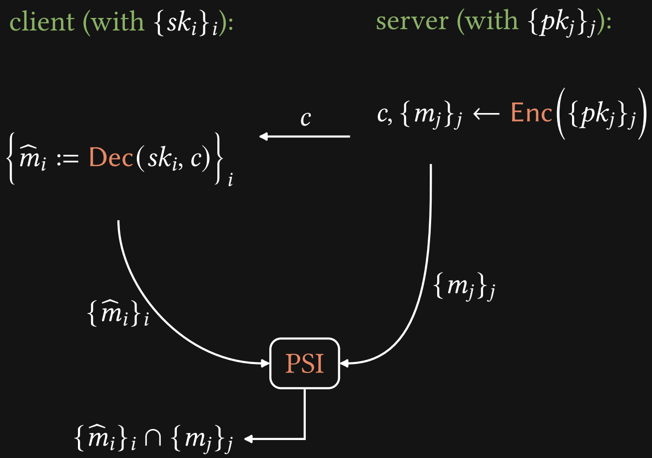

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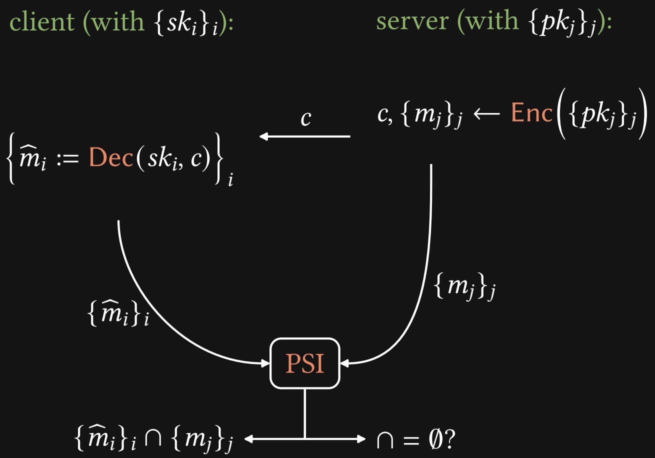

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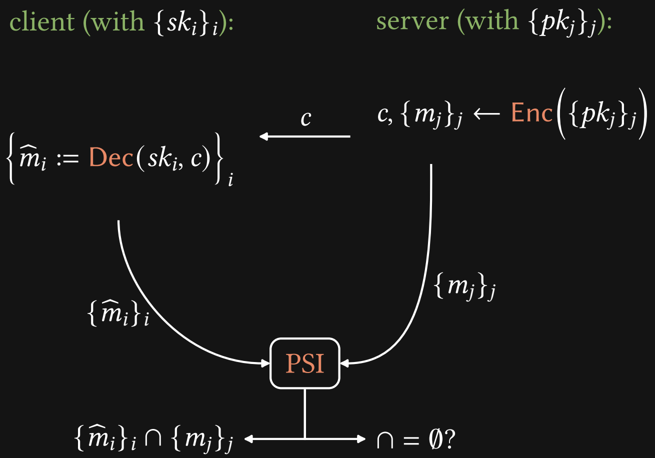

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single MKEM construction supporting RSA, ECDSA, & EdDSA

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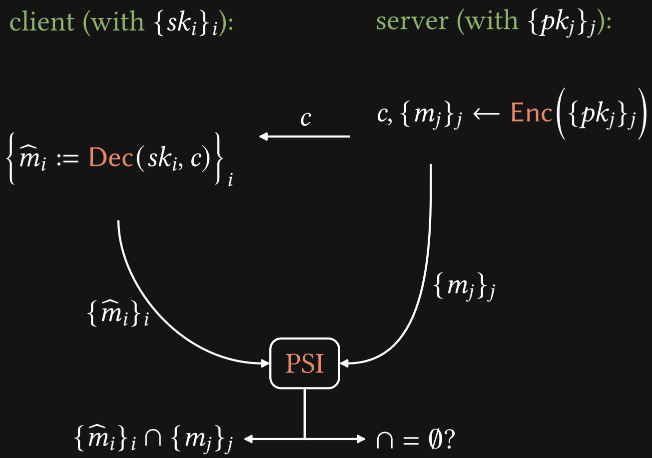

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add "proof of nonempty intersection" to [RosulekTrieu21] PSI

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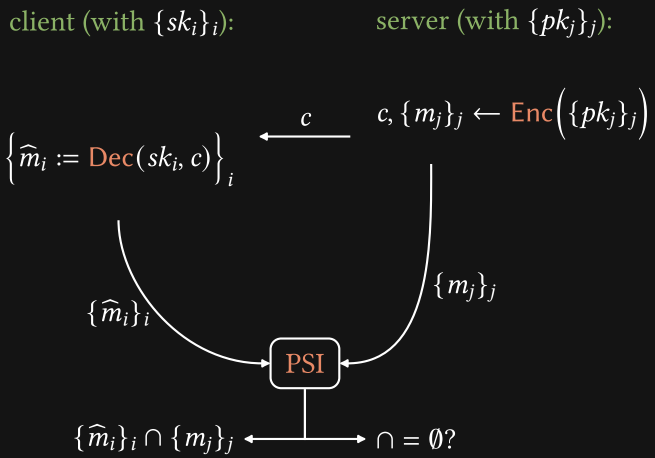

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+ full UC security analysis

concrete performance (in OpenSSH):

# of keys		RSA keys only (worst case for us)		{EC,Ed}DSA keys only (best case for us)	
client	server	time	comm	time	comm

`github.com/osu-crypto/PSIPK-ssh`

2 commodity desktop computers on LAN

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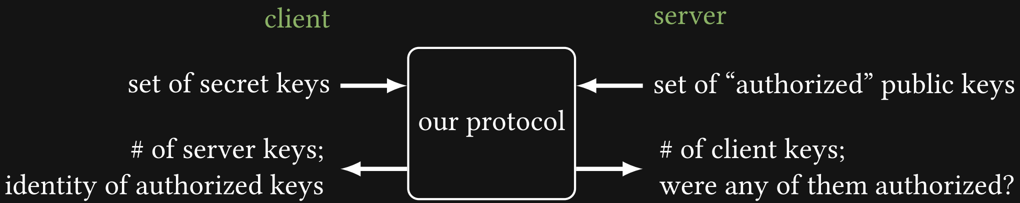
2 commodity desktop computers on LAN



client

server

set of secret keys

our protocol

set of “authorized” public keys

# of server keys;
identity of authorized keys

# of client keys;
were any of them authorized?

- ✓ efficient, practical
- ✓ mixture of existing RSA & EC keys
- ✓ safe without special per-site configuration

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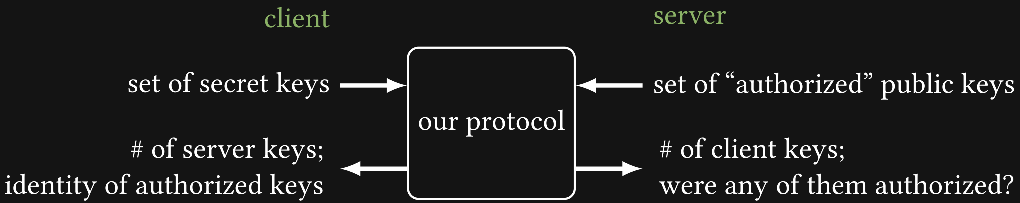
`ia.cr/2022/740`



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thanks!

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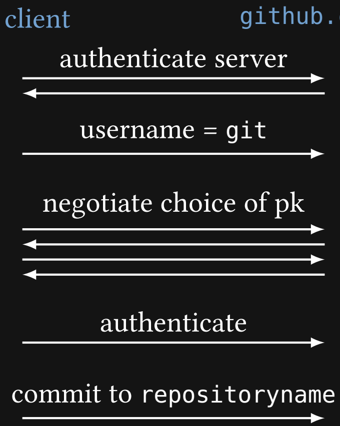
ia.cr/2022/740

(backup slides)

github over SSH:



client `github.com`

authenticate server

username = `git`

negotiate choice of pk

authenticate

commit to `repositoryname`

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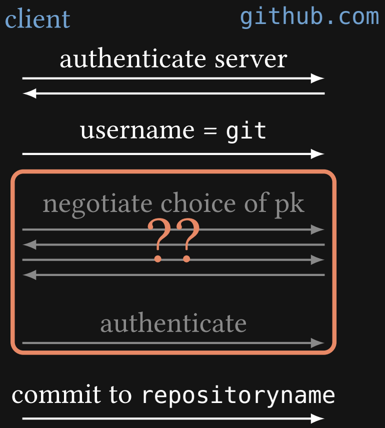

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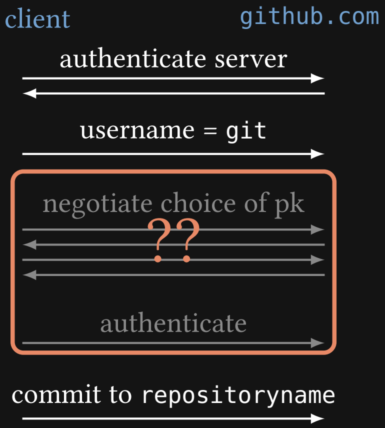

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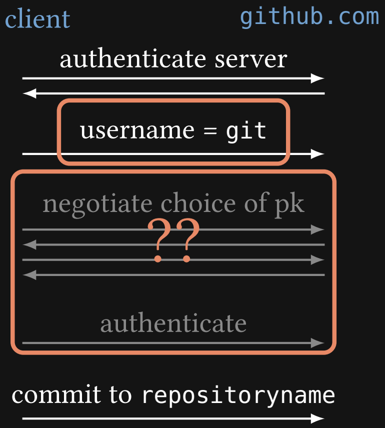

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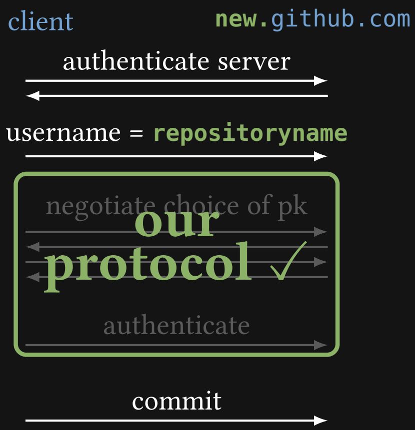

- server must decide **set of authorized keys** before running our protocol!
- server does not know repository name yet!
- use repository name as `username`

anonymous multi-KEM

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address ciphertext to $\{pk_j\}_j$;
sk_j decrypts c to m_j;
c hides pk_j recipients

the case of EdDSA/ECDSA

Alice: $pk_A = g^a$

Bob: $pk_B = g^b$

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$\xleftarrow{\text{ciphertext} = g^r}$

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ciphertext hides set of recipients; even # of them!

the case of RSA

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encrypt $(r_A)^{e_A} \bmod N_A$

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interpolate poly P:

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PSI with proof of nonempty intersection

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each party has set of items;
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oblivious PRF (OPRF) paradigm for PSI

[FreedmanIshaiPinkasReingold05]

Alice:

$X = \{x_1, x_2, \ldots\}$

Bob:

$Y = \{y_1, y_2, \ldots\}$

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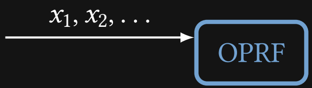

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random $F(\cdot)$

$F(x_1), F(x_2), \ldots$

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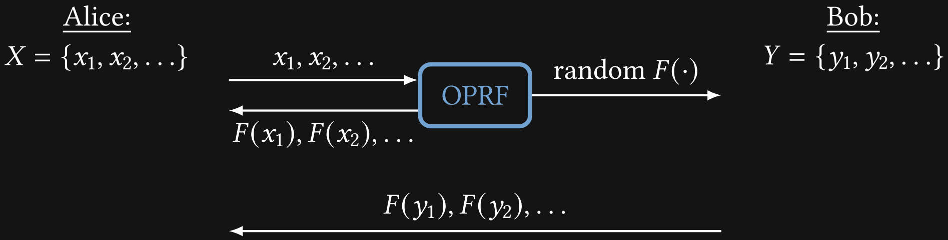

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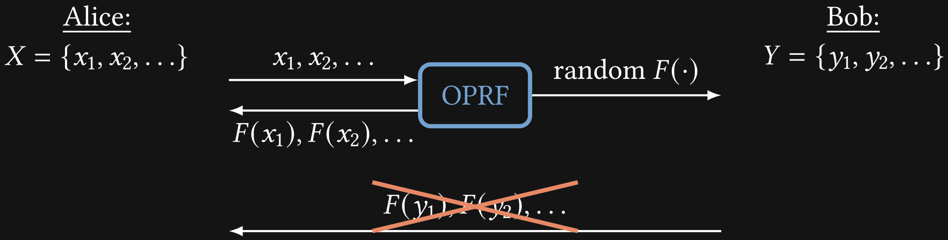

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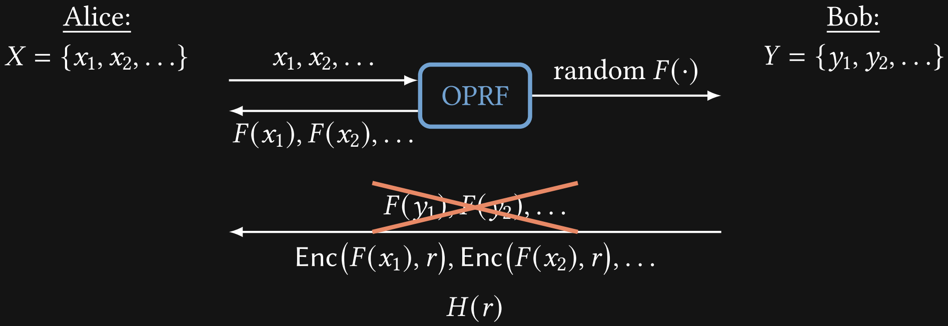

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$H(r)$

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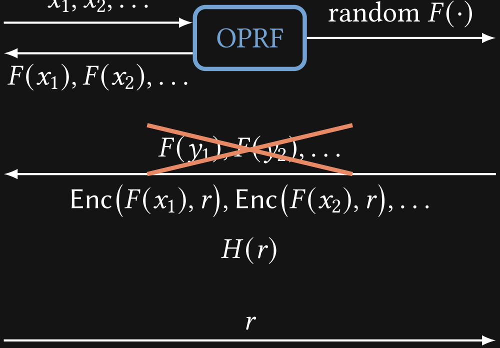

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